THE CASE $r = 2$, $m = 4$

We attach the Mathematica prints that we use in Sect. 4 and in the Appendix. Here is an itemized list to guide the reader:

(1) Out[3]–Out[14] computes the expression $f_{a,4,4,2,0}$ given in (A.5).
(2) Out[15]–Out[24] provides the expression of $f_{a,4,s,2,0}$ given in Lemma A.4(i).
(3) Out[26] verifies Lemma 4.1(iv) and (vi).
(4) Out[27] verifies (4.3), (4.4).
(5) Out[29] verifies (4.8).
(6) Out[31] verifies (4.7).
(7) Out[33] verifies the formula for $\chi(\mathcal{O}_Z)$ given after (4.8).
(8) Out[34] verifies Lemma A.5(1).
(9) Out[37] verifies Lemma A.6(i).

```
(*We calculate (360/(x1x2x3x4))f_{a,4,4,2,0} in variables
{a,x1,x2,x3,x4}. This is symmetric in {x1,x2,x3,x4}*)
```

In[1]:=

```
FunctionExpand[1 - (1 / 12) * x1 * x2 * x3 * x4 * (5 * a + x1 + x2 + x3 + x4 - 9 - a) * (5 * a + x1 + x2 + x3 +
  x4 - 9 - 2 * a) * (5 * a + x1 + x2 + x3 + x4 - 9 - 3 * a) * (5 * a + x1 + x2 + x3 + x4 - 9 - 4 * a) + Binomial[
  5 * a + x1 + x2 + x3 + x4 - 10, 8] - Binomial[x1 - 1, 8] - Binomial[x2 - 1, 8] - Binomial[x3 - 1,
  8] - Binomial[x4 - 1, 8] - Binomial[x1 + 5 * a + x1 + x2 + x3 + x4 - 10, 8] - Binomial[x2 + 5 * a +
  x1 + x2 + x3 + x4 - 10, 8] - Binomial[x3 + 5 * a + x1 + x2 + x3 + x4 - 10, 8] - Binomial[x4 + 5 *
  a + x1 + x2 + x3 + x4 - 10, 8] + Binomial[x1 + x2 - 1, 8] + Binomial[x1 + x3 - 1, 8] + Binomial[
  x1 + x4 - 1, 8] + Binomial[x2 + x3 - 1, 8] + Binomial[x2 + x4 - 1, 8] + Binomial[x3 + x4 - 1,
  8] + Binomial[x1 + x2 + 5 * a + x1 + x2 + x3 + x4 - 10, 8] + Binomial[x1 + x3 + 5 * a + x1 + x2 + x3 +
  x4 - 10, 8] + Binomial[x1 + x4 + 5 * a + x1 + x2 + x3 + x4 - 10, 8] + Binomial[x2 + x3 + 5 * a + x1 +
  x2 + x3 + x4 - 10, 8] + Binomial[x2 + x4 + 5 * a + x1 + x2 + x3 + x4 - 10, 8] + Binomial[x3 + x4 +
  5 * a + x1 + x2 + x3 + x4 - 10, 8] - Binomial[x1 + x2 + x3 - 1, 8] - Binomial[x1 + x2 + x4 - 1, 8] -
  Binomial[x1 + x3 + x4 - 1, 8] - Binomial[x2 + x3 + x4 - 1, 8] - Binomial[x1 + x2 + x3 + 5 * a + x1 +
  x2 + x3 + x4 - 10, 8] - Binomial[x1 + x2 + x4 + 5 * a + x1 + x2 + x3 + x4 - 10, 8] - Binomial[x2 +
  x3 + x4 + 5 * a + x1 + x2 + x3 + x4 - 10, 8] - Binomial[x1 + x3 + x4 + 5 * a + x1 + x2 + x3 + x4 - 10, 8] +
  Binomial[x1 + x2 + x3 + x4 - 1, 8] + Binomial[x1 + x2 + x3 + x4 + 5 * a + x1 + x2 + x3 + x4 - 10, 8]]
```

Out[1]=

```
1 - (-8+x1) (-7+x1) (-6+x1) (-5+x1) (-4+x1) (-3+x1) (-2+x1) (-1+x1) / 40 320 - (-8+x2) ⋯6⋯ (⋯1⋯) / 40 320 + ⋯39⋯ + ⋯1⋯ -
 ⋯1⋯ / 40 320 - (-17+5 a+x1+2 x2+2 x3+2 x4) ⋯6⋯ (-10+5 a+x1+ ⋯1⋯ +2 x3+2 x4) / 40 320 + 1/40 320 (-17 + 5 a + 2 x1 + 2 x2 + 2 x3 + 2 x4)
 (-16 + 5 a + 2 x1 + 2 x2 + 2 x3 + 2 x4) (-15 + 5 a + 2 x1 + 2 x2 + 2 x3 + 2 x4) (-14 + ⋯4⋯ + 2 x4) ⋯1⋯
 (-12 + 5 a + 2 x1 + 2 x2 + 2 x3 + 2 x4) (-11 + 5 a + 2 x1 + 2 x2 + 2 x3 + 2 x4) (-10 + 5 a + 2 x1 + 2 x2 + 2 x3 + 2 x4)
```

Full expression not available (original memory size: 57 kB)

In[2]:=
```
Expand[360 / (x1 * x2 * x3 * x4) * %1]
```

Out[2]=

$$293\,931 - 510\,300\, a + 323\,325\, a^2 - 87\,750\, a^3 + 8655\, a^4 - 133\,650\, x1 + 172\,125\, a\, x1 - 72\,225\, a^2\, x1 +$$
$$9750\, a^3\, x1 + 24\,120\, x1^2 - 20\,250\, a\, x1^2 + 4200\, a^2\, x1^2 - 2025\, x1^3 + 825\, a\, x1^3 + 66\, x1^4 - 133\,650\, x2 +$$
$$172\,125\, a\, x2 - 72\,225\, a^2\, x2 + 9750\, a^3\, x2 + 45\,225\, x1\, x2 - 38\,475\, a\, x1\, x2 + 8025\, a^2\, x1\, x2 -$$
$$5400\, x1^2\, x2 + 2250\, a\, x1^2\, x2 + 225\, x1^3\, x2 + 24\,120\, x2^2 - 20\,250\, a\, x2^2 + 4200\, a^2\, x2^2 - 5400\, x1\, x2^2 +$$
$$2250\, a\, x1\, x2^2 + 320\, x1^2\, x2^2 - 2025\, x2^3 + 825\, a\, x2^3 + 225\, x1\, x2^3 + 66\, x2^4 - 133\,650\, x3 + 172\,125\, a\, x3 -$$
$$72\,225\, a^2\, x3 + 9750\, a^3\, x3 + 45\,225\, x1\, x3 - 38\,475\, a\, x1\, x3 + 8025\, a^2\, x1\, x3 - 5400\, x1^2\, x3 +$$
$$2250\, a\, x1^2\, x3 + 225\, x1^3\, x3 + 45\,225\, x2\, x3 - 38\,475\, a\, x2\, x3 + 8025\, a^2\, x2\, x3 - 10\,125\, x1\, x2\, x3 +$$
$$4275\, a\, x1\, x2\, x3 + 600\, x1^2\, x2\, x3 - 5400\, x2^2\, x3 + 2250\, a\, x2^2\, x3 + 600\, x1\, x2^2\, x3 + 225\, x2^3\, x3 +$$
$$24\,120\, x3^2 - 20\,250\, a\, x3^2 + 4200\, a^2\, x3^2 - 5400\, x1\, x3^2 + 2250\, a\, x1\, x3^2 + 320\, x1^2\, x3^2 - 5400\, x2\, x3^2 +$$
$$2250\, a\, x2\, x3^2 + 600\, x1\, x2\, x3^2 + 320\, x2^2\, x3^2 - 2025\, x3^3 + 825\, a\, x3^3 + 225\, x1\, x3^3 + 225\, x2\, x3^3 +$$
$$66\, x3^4 - 133\,650\, x4 + 172\,125\, a\, x4 - 72\,225\, a^2\, x4 + 9750\, a^3\, x4 + 45\,225\, x1\, x4 - 38\,475\, a\, x1\, x4 +$$
$$8025\, a^2\, x1\, x4 - 5400\, x1^2\, x4 + 2250\, a\, x1^2\, x4 + 225\, x1^3\, x4 + 45\,225\, x2\, x4 - 38\,475\, a\, x2\, x4 +$$
$$8025\, a^2\, x2\, x4 - 10\,125\, x1\, x2\, x4 + 4275\, a\, x1\, x2\, x4 + 600\, x1^2\, x2\, x4 - 5400\, x2^2\, x4 + 2250\, a\, x2^2\, x4 +$$
$$600\, x1\, x2^2\, x4 + 225\, x2^3\, x4 + 45\,225\, x3\, x4 - 38\,475\, a\, x3\, x4 + 8025\, a^2\, x3\, x4 - 10\,125\, x1\, x3\, x4 +$$
$$4275\, a\, x1\, x3\, x4 + 600\, x1^2\, x3\, x4 - 10\,125\, x2\, x3\, x4 + 4275\, a\, x2\, x3\, x4 + 1125\, x1\, x2\, x3\, x4 +$$
$$600\, x2^2\, x3\, x4 - 5400\, x3^2\, x4 + 2250\, a\, x3^2\, x4 + 600\, x1\, x3^2\, x4 + 600\, x2\, x3^2\, x4 + 225\, x3^3\, x4 +$$
$$24\,120\, x4^2 - 20\,250\, a\, x4^2 + 4200\, a^2\, x4^2 - 5400\, x1\, x4^2 + 2250\, a\, x1\, x4^2 + 320\, x1^2\, x4^2 - 5400\, x2\, x4^2 +$$
$$2250\, a\, x2\, x4^2 + 600\, x1\, x2\, x4^2 + 320\, x2^2\, x4^2 - 5400\, x3\, x4^2 + 2250\, a\, x3\, x4^2 + 600\, x1\, x3\, x4^2 +$$
$$600\, x2\, x3\, x4^2 + 320\, x3^2\, x4^2 - 2025\, x4^3 + 825\, a\, x4^3 + 225\, x1\, x4^3 + 225\, x2\, x4^3 + 225\, x3\, x4^3 + 66\, x4^4$$

```
(*We calculate all the coefficients of the monomial symmetric
polynomials in {x1,x2,x3,x4} appearing in (360/(x1x2x3x4))f_{a,4,4,2,0}*)
```

In[3]:=
```
a1 = SeriesCoefficient[%2, {x1, 0, 4}, {x2, 0, 0}, {x3, 0, 0}, {x4, 0, 0}]
```

Out[3]= 66

In[4]:=
```
a2 = SeriesCoefficient[%2, {x1, 0, 3}, {x2, 0, 1}, {x3, 0, 0}, {x4, 0, 0}]
```

Out[4]= 225

In[5]:=
```
a3 = SeriesCoefficient[%2, {x1, 0, 2}, {x2, 0, 2}, {x3, 0, 0}, {x4, 0, 0}]
```

Out[5]= 320

In[6]:=
```
a4 = SeriesCoefficient[%2, {x1, 0, 2}, {x2, 0, 1}, {x3, 0, 1}, {x4, 0, 0}]
```

Out[6]= 600

In[7]:=
```
a5 = SeriesCoefficient[%2, {x1, 0, 1}, {x2, 0, 1}, {x3, 0, 1}, {x4, 0, 1}]
```

Out[7]= 1125

In[8]:=
```
l6 = SeriesCoefficient[%2, {x1, 0, 3}, {x2, 0, 0}, {x3, 0, 0}, {x4, 0, 0}]
```

Out[8]= $-2025 + 825\, a$

In[9]:=
```
l7 = SeriesCoefficient[%2, {x1, 0, 2}, {x2, 0, 1}, {x3, 0, 0}, {x4, 0, 0}]
```

Out[9]= $-5400 + 2250\, a$

In[10]:=
```
l8 = SeriesCoefficient[%2, {x1, 0, 1}, {x2, 0, 1}, {x3, 0, 1}, {x4, 0, 0}]
```
Out[10]=

$-10\,125 + 4275\,a$

In[11]:=
```
l9 = SeriesCoefficient[%2, {x1, 0, 2}, {x2, 0, 0}, {x3, 0, 0}, {x4, 0, 0}]
```
Out[11]=

$24\,120 - 20\,250\,a + 4200\,a^2$

In[12]:=
```
l10 = SeriesCoefficient[%2, {x1, 0, 1}, {x2, 0, 1}, {x3, 0, 0}, {x4, 0, 0}]
```
Out[12]=

$45\,225 - 38\,475\,a + 8025\,a^2$

In[13]:=
```
l11 = SeriesCoefficient[%2, {x1, 0, 1}, {x2, 0, 0}, {x3, 0, 0}, {x4, 0, 0}]
```
Out[13]=

$-133\,650 + 172\,125\,a - 72\,225\,a^2 + 9750\,a^3$

In[14]:=
```
l12 = SeriesCoefficient[%2, {x1, 0, 0}, {x2, 0, 0}, {x3, 0, 0}, {x4, 0, 0}]
```
Out[14]=

$293\,931 - 510\,300\,a + 323\,325\,a^2 - 87\,750\,a^3 + 8655\,a^4$

```
(*We calculate all the coefficients of the monomial symmetric polynomials in {x_
1,...,x_s} appearing in (360/(product x_i))f_{a,4,s,2,0}* using Lemma A.6* of [LR2])
```

In[15]:=
```
a6 = Expand[l6 - (s - 4) * (a2)]
```
Out[15]=

$-1125 + 825\,a - 225\,s$

In[16]:=
```
a7 = Expand[l7 - (s - 4) * (a4)]
```
Out[16]=

$-3000 + 2250\,a - 600\,s$

In[17]:=
```
a8 = Expand[l8 - (s - 4) * (a5)]
```
Out[17]=

$-5625 + 4275\,a - 1125\,s$

In[18]:=
```
a9 = Expand[l9 - (s - 4) * (a3 + a7) - Binomial[s - 4, 2] * (a4)]
```
Out[18]=

$7400 - 11\,250\,a + 4200\,a^2 + 2980\,s - 2250\,a\,s + 300\,s^2$

In[19]:=
```
Expand[l10 - (s - 4) * (a4 + a8) - Binomial[s - 4, 2] * (a5)]
```
Out[19]=

$$13\,875 - 21\,375\,a + 8025\,a^2 + \frac{11\,175\,s}{2} - 4275\,a\,s + \frac{1125\,s^2}{2}$$

In[20]:=
```
a10 = Factor[%19]
```
Out[20]=

$$\frac{75}{2}\left(370 - 570\,a + 214\,a^2 + 149\,s - 114\,a\,s + 15\,s^2\right)$$

```
In[21]:= Expand[l11 - (s - 4) * (a2 + a7 + a10) - Binomial[s - 4, 2] * (2 * a4 + a8) - Binomial[s - 4, 3] * (a5)]
```

Out[21]=

$$-22500 + 52875 a - 40125 a^2 + 9750 a^3 - \frac{27375 s}{2} + \frac{42525 a s}{2} - 8025 a^2 s - 2775 s^2 + \frac{4275 a s^2}{2} - \frac{375 s^3}{2}$$

```
In[22]:= a11 = Factor[%21]
```

Out[22]=

$$\frac{75}{2} \left(-600 + 1410 a - 1070 a^2 + 260 a^3 - 365 s + 567 a s - 214 a^2 s - 74 s^2 + 57 a s^2 - 5 s^3\right)$$

```
In[23]:= Expand[l12 - (s - 4) * (a1 + a6 + a9 + a11) - Binomial[s - 4, 2] * (2 * a2 +
          a3 + 2 * a7 + a10) - Binomial[s - 4, 3] * (3 * a4 + a8) - Binomial[s - 4, 4] * (a5)]
```

Out[23]=

$$26970 - 86250 a + 99375 a^2 - 48750 a^3 + 8655 a^4 + \frac{88151 s}{4} - \frac{104625 a s}{2} + \frac{79875 a^2 s}{2} - 9750 a^3 s + \frac{54005 s^2}{8} - 10575 a s^2 + \frac{8025 a^2 s^2}{2} + \frac{3675 s^3}{4} - \frac{1425 a s^3}{2} + \frac{375 s^4}{8}$$

```
In[24]:= a12 = Factor[%23]
```

Out[24]=

$$\frac{1}{8} \left(215760 - 690000 a + 795000 a^2 - 390000 a^3 + 69240 a^4 + 176302 s - 418500 a s + 319500 a^2 s - 78000 a^3 s + 54005 s^2 - 84600 a s^2 + 32100 a^2 s^2 + 7350 s^3 - 5700 a s^3 + 375 s^4\right)$$

```
(*Defining the functions computing (1/(product x_i))f_{a,4,s,2,0}*)
```

```
In[25]:= Fa4s20 = (1 / 360) * (a1 * m4 + a2 * m31 + a3 * m22 + a4 * m211 + a5 *
          m1111 + a6 * m3 + a7 * m21 + a8 * m111 + a9 * m2 + a10 * m11 + a11 * m1 + a12)
```

Out[25]=

$$\frac{1}{360} \Big(1125\, m1111 + 600\, m211 + 320\, m22 + 225\, m31 + 66\, m4 + m111 (-5625 + 4275 a - 1125 s) + m21 (-3000 + 2250 a - 600 s) + m3 (-1125 + 825 a - 225 s) + \frac{75}{2} m11 (370 - 570 a + 214 a^2 + 149 s - 114 a s + 15 s^2) + m2 (7400 - 11250 a + 4200 a^2 + 2980 s - 2250 a s + 300 s^2) + \frac{75}{2} m1 (-600 + 1410 a - 1070 a^2 + 260 a^3 - 365 s + 567 a s - 214 a^2 s - 74 s^2 + 57 a s^2 - 5 s^3) + \frac{1}{8} (215760 - 690000 a + 795000 a^2 - 390000 a^3 + 69240 a^4 + 176302 s - 418500 a s + 319500 a^2 s - 78000 a^3 s + 54005 s^2 - 84600 a s^2 + 32100 a^2 s^2 + 7350 s^3 - 5700 a s^3 + 375 s^4)\Big)$$

```
(*We compute the polynomial calculating (24/(rd))deg(Z)
appearing in Lemma 4.1(iv) where X is in P^{m+s}, c.i. of
type (d_1,...,d_s), E Ulrich for (X,O_X(a)) using Lemma 2.2(ii)*)
```

In[26]:=
```
Expand[(24 / (r * d)) ((1 / 2) * ((r / 2) * ((m + 1) * (a - 1) + m1 - s)) ^ 2 * d - (1 / 2) * ((r / 2) *
  ((m + 1) * (a - 1) + m1 - s)) * (m1 - s - m - 1) * d + (r / 12) * (m1 - s - m - 1) ^ 2 * d + (r * d / 12) *
  (Binomial[m + s + 1, 2] + m1 * (m1 - s - m - 1) - m11) - (r * d / 24) * a ^ 2 * (3 * m ^ 2 + 5 * m + 2))]
```

Out[26]=

$$-4 + 6 a - 2 a^2 - 7 m + 12 a m - 5 a^2 m - 3 m^2 + 6 a m^2 - 3 a^2 m^2 + 6 m1 - 6 a m1 + 6 m m1 - 6 a m m1 - 2 m1^2 - 2 m11 + 3 r - 6 a r + 3 a^2 r + 6 m r - 12 a m r + 6 a^2 m r + 3 m^2 r - 6 a m^2 r + 3 a^2 m^2 r - 6 m1 r + 6 a m1 r - 6 m m1 r + 6 a m m1 r + 3 m1^2 r - 7 s + 6 a s - 6 m s + 6 a m s + 6 m1 s + 6 r s - 6 a r s + 6 m r s - 6 a m r s - 6 m1 r s - 3 s^2 + 3 r s^2$$

```
(*Specializing the above when m=4,r=2*)
```

In[27]:=
```
d1 = %26 /. {r → 2, m → 4}
```

Out[27]=

$$70 - 150 a + 80 a^2 - 30 m1 + 30 a m1 + 4 m1^2 - 2 m11 + 29 s - 30 a s - 6 m1 s + 3 s^2$$

```
(*We compute the polynomial calculating (12c_2(Z))/deg(Z) where X is in P^{4+s},
c.i. of type (d_1,...,d_s), E Ulrich of rank 2 for (X,O_X(a)) using Lemma 3.1(ii)*)
```

In[28]:=
```
FunctionExpand[Binomial[s + 5, 2] + m1 * (m1 - s -
  5) - m11 - (1 / 12) * d1 + (2 * m1 - 2 * s + 5 * (a - 2)) * (m1 - s + 5 * (a - 1))]
```

Out[28]=

$$-m11 + m1 (-5 + m1 - s) + (5 (-2 + a) + 2 m1 - 2 s) (5 (-1 + a) + m1 - s) + \frac{1}{2} (4 + s) (5 + s) + \frac{1}{12} (-70 + 150 a - 80 a^2 + 30 m1 - 30 a m1 - 4 m1^2 + 2 m11 - 29 s + 30 a s + 6 m1 s - 3 s^2)$$

In[29]:=
```
d2 = Expand[12 * %28]
```

Out[29]=

$$650 - 750 a + 220 a^2 - 270 m1 + 150 a m1 + 32 m1^2 - 10 m11 + 265 s - 150 a s - 54 m1 s + 27 s^2$$

```
(*We compute the polynomial calculating (144/d)c_2(Z) where X is in P^
{4+s}, c.i. of type (d_1,...,d_s), E Ulrich of rank 2 for (X,O_X(a))*)
```

In[30]:=
```
p1 = Expand[d2 * d1]
```

Out[30]=

$$45500 - 150000 a + 179900 a^2 - 93000 a^3 + 17600 a^4 - 38400 m1 + 93000 a m1 - 73200 a^2 m1 + 18600 a^3 m1 + 12940 m1^2 - 20400 a m1^2 + 7940 a^2 m1^2 - 2040 m1^3 + 1560 a m1^3 + 128 m1^4 - 2000 m11 + 3000 a m11 - 1240 a^2 m11 + 840 m1 m11 - 600 a m1 m11 - 104 m1^2 m11 + 20 m11^2 + 37400 s - 91500 a s + 72580 a^2 s - 18600 a^3 s - 23460 m1 s + 37500 a m1 s - 14640 a^2 m1 s + 5228 m1^2 s - 4080 a m1^2 s - 408 m1^3 s - 820 m11 s + 600 a m11 s + 168 m1 m11 s + 11525 s^2 - 18600 a s^2 + 7320 a^2 s^2 - 4776 m1 s^2 + 3780 a m1 s^2 + 528 m1^2 s^2 - 84 m11 s^2 + 1578 s^3 - 1260 a s^3 - 324 m1 s^3 + 81 s^4$$

```
(*We compute the polynomial calculating K_Z^2/deg(Z) where X is in P^
{4+s}, c.i. of type (d_1,...,d_s), E Ulrich of rank 2 for (X,O_X(a))*)
```

In[31]:=
```
d3 = Expand[(2 * m1 - 2 * s + 5 * (a - 2)) ^ 2]
```

Out[31]=

$$100 - 100 a + 25 a^2 - 40 m1 + 20 a m1 + 4 m1^2 + 40 s - 20 a s - 8 m1 s + 4 s^2$$

```
(*We compute the polynomial calculating (12/d)K_Z^2 where X is in P^
{4+s}, c.i. of type (d_1,...,d_s), E Ulrich of rank 2 for (X,O_X(a))*)
```

In[32]:=
```
p2 = Expand[d3 * d1]
```

Out[32]=

$$7000 - 22\,000\, a + 24\,750\, a^2 - 11\,750\, a^3 + 2000\, a^4 - 5800\, m1 + 13\,400\, a\, m1 - 9950\, a^2\, m1 + 2350\, a^3\, m1 +$$
$$1880\, m1^2 - 2800\, a\, m1^2 + 1020\, a^2\, m1^2 - 280\, m1^3 + 200\, a\, m1^3 + 16\, m1^4 - 200\, m11 + 200\, a\, m11 - 50\, a^2\, m11 +$$
$$80\, m1\, m11 - 40\, a\, m1\, m11 - 8\, m1^2\, m11 + 5700\, s - 13\,300\, a\, s + 9925\, a^2\, s - 2350\, a^3\, s - 3520\, m1\, s + 5380\, a\, m1\, s -$$
$$1990\, a^2\, m1\, s + 756\, m1^2\, s - 560\, a\, m1^2\, s - 56\, m1^3\, s - 80\, m11\, s + 40\, a\, m11\, s + 16\, m1\, m11\, s + 1740\, s^2 -$$
$$2680\, a\, s^2 + 995\, a^2\, s^2 - 712\, m1\, s^2 + 540\, a\, m1\, s^2 + 76\, m1^2\, s^2 - 8\, m11\, s^2 + 236\, s^3 - 180\, a\, s^3 - 48\, m1\, s^3 + 12\, s^4$$

```
(*We compute the polynomial calculating (144/(5d))(K_Z^2+c_2(Z)) where X is
in P^{4+s}, c.i. of type (d_1,...,d_s), E Ulrich of rank 2 for (X,O_X(a))*)
```

In[33]:=
```
f1 = Expand[(1 / 5) * (12 * p2 + p1)]
```

Out[33]=

$$25\,900 - 82\,800\, a + 95\,380\, a^2 - 46\,800\, a^3 + 8320\, a^4 - 21\,600\, m1 + 50\,760\, a\, m1 - 38\,520\, a^2\, m1 +$$
$$9360\, a^3\, m1 + 7100\, m1^2 - 10\,800\, a\, m1^2 + 4036\, a^2\, m1^2 - 1080\, m1^3 + 792\, a\, m1^3 + 64\, m1^4 - 880\, m11 +$$
$$1080\, a\, m11 - 368\, a^2\, m11 + 360\, m1\, m11 - 216\, a\, m1\, m11 - 40\, m1^2\, m11 + 4\, m11^2 + 21\,160\, s - 50\,220\, a\, s +$$
$$38\,336\, a^2\, s - 9360\, a^3\, s - 13\,140\, m1\, s + 20\,412\, a\, m1\, s - 7704\, a^2\, m1\, s + 2860\, m1^2\, s - 2160\, a\, m1^2\, s -$$
$$216\, m1^3\, s - 356\, m11\, s + 216\, a\, m11\, s + 72\, m1\, m11\, s + 6481\, s^2 - 10\,152\, a\, s^2 + 3852\, a^2\, s^2 -$$
$$2664\, m1\, s^2 + 2052\, a\, m1\, s^2 + 288\, m1^2\, s^2 - 36\, m11\, s^2 + 882\, s^3 - 684\, a\, s^3 - 180\, m1\, s^3 + 45\, s^4$$

In[34]:=
```
f2 = Expand[%33 /. {(m1)^2 → m2 + 2 * m11, (m1)^3 → m3 + 3 * m21 + 6 * m111, (m1)^4 → m4 +
  4 * m31 + 6 * m22 + 12 * m211 + 24 * m1111, m1 * m11 → m21 + 3 * m111, (m1)^2 * m11 → m31 +
  2 * m22 + 5 * m211 + 12 * m1111, (m11)^2 → m22 + 2 * m211 + 6 * m1111, m1 * m3 → m4 + m31,
  m1 * m21 → m31 + 2 * m22 + 2 * m211, m1 * m111 → m211 + 4 * m1111, m1 * m2 → m3 + m21, (m1)^
  2 * m2 → m4 + 2 * m31 + 2 * m22 + 2 * m211, (m2)^2 → m4 + 2 * m22, m2 * m11 → m31 + m211}]
```

Out[34]=

$$25\,900 - 82\,800\, a + 95\,380\, a^2 - 46\,800\, a^3 + 8320\, a^4 - 21\,600\, m1 + 50\,760\, a\, m1 - 38\,520\, a^2\, m1 + 9360\, a^3\, m1 +$$
$$13\,320\, m11 - 20\,520\, a\, m11 + 7704\, a^2\, m11 - 5400\, m111 + 4104\, a\, m111 + 1080\, m1111 + 7100\, m2 - 10\,800\, a\, m2 +$$
$$4036\, a^2\, m2 - 2880\, m21 + 2160\, a\, m21 + 576\, m211 + 308\, m22 - 1080\, m3 + 792\, a\, m3 + 216\, m31 + 64\, m4 +$$
$$21\,160\, s - 50\,220\, a\, s + 38\,336\, a^2\, s - 9360\, a^3\, s - 13\,140\, m1\, s + 20\,412\, a\, m1\, s - 7704\, a^2\, m1\, s + 5364\, m11\, s -$$
$$4104\, a\, m11\, s - 1080\, m111\, s + 2860\, m2\, s - 2160\, a\, m2\, s - 576\, m21\, s - 216\, m3\, s + 6481\, s^2 - 10\,152\, a\, s^2 +$$
$$3852\, a^2\, s^2 - 2664\, m1\, s^2 + 2052\, a\, m1\, s^2 + 540\, m11\, s^2 + 288\, m2\, s^2 + 882\, s^3 - 684\, a\, s^3 - 180\, m1\, s^3 + 45\, s^4$$

```
(*Main relation when X is in P^{4+s}, c.i. of type (
d_1,...,d_s), E Ulrich of rank 2 for (X,O_X(a))*)
```

In[35]:=
```
Ga4s = (5 / 1728) * f2
```

Out[35]=

$$\frac{1}{1728} 5 \left(25\,900 - 82\,800\, a + 95\,380\, a^2 - 46\,800\, a^3 + 8320\, a^4 - 21\,600\, m1 + 50\,760\, a\, m1 - 38\,520\, a^2\, m1 + 9360\, a^3\, m1 + 13\,320\, m11 - 20\,520\, a\, m11 + 7704\, a^2\, m11 - 5400\, m111 + 4104\, a\, m111 + 1080\, m1111 + 7100\, m2 - 10\,800\, a\, m2 + 4036\, a^2\, m2 - 2880\, m21 + 2160\, a\, m21 + 576\, m211 + 308\, m22 - 1080\, m3 + 792\, a\, m3 + 216\, m31 + 64\, m4 + 21\,160\, s - 50\,220\, a\, s + 38\,336\, a^2\, s - 9360\, a^3\, s - 13\,140\, m1\, s + 20\,412\, a\, m1\, s - 7704\, a^2\, m1\, s + 5364\, m11\, s - 4104\, a\, m11\, s - 1080\, m111\, s + 2860\, m2\, s - 2160\, a\, m2\, s - 576\, m21\, s - 216\, m3\, s + 6481\, s^2 - 10\,152\, a\, s^2 + 3852\, a^2\, s^2 - 2664\, m1\, s^2 + 2052\, a\, m1\, s^2 + 540\, m11\, s^2 + 288\, m2\, s^2 + 882\, s^3 - 684\, a\, s^3 - 180\, m1\, s^3 + 45\, s^4\right)$$

In[36]:=
```
Expand[Ga4s - Fa4s20]
```

Out[36]=

$$\frac{11}{432} - \frac{25\, a^2}{432} + \frac{7\, a^4}{216} - \frac{5\, m2}{432} + \frac{5\, a^2\, m2}{432} + \frac{m22}{432} + \frac{m4}{540} + \frac{47\, s}{4320} - \frac{5\, a^2\, s}{432} - \frac{m2\, s}{432} + \frac{s^2}{864}$$

In[37]:=

$$\text{Factor}\left[\frac{11}{432} - \frac{25\, a^2}{432} + \frac{7\, a^4}{216} - \frac{5\, m2}{432} + \frac{5\, a^2\, m2}{432} + \frac{m22}{432} + \frac{m4}{540} + \frac{47\, s}{4320} - \frac{5\, a^2\, s}{432} - \frac{m2\, s}{432} + \frac{s^2}{864}\right]$$

Out[37]=

$$\frac{110 - 250\, a^2 + 140\, a^4 - 50\, m2 + 50\, a^2\, m2 + 10\, m22 + 8\, m4 + 47\, s - 50\, a^2\, s - 10\, m2\, s + 5\, s^2}{4320}$$

THE CASE $r = 3$, $m = 4$

We attach the Mathematica prints that we use in Sect. 4 and in the Appendix. Here is an itemized list to guide the reader:

(1) Out[3]–Out[14] computes the expression $f_{a,4,4,3,0}$ given in (A.6).
(2) Out[15]–Out[21] provides the expression of $f_{a,4,s,3,0}$ given in Lemma A.4(ii).
(3) Out[24]–Out[35] computes the expression $f_{a,4,4,3,1}$ given in (A.7).
(4) Out[36]–Out[42] provides the expression of $f_{a,4,s,3,1}$ given in Lemma A.4(iii).
(5) Out[48] verifies the expression of $\deg_H(Z)$ given after (4.11).
(6) Out[50] verifies Lemma A.5(2).
(7) Out[51] verifies Lemma A.5(3).
(8) Out[60] verifies Lemma A.5(4).
(9) Out[70] verifies Lemma A.5(5).
(10) Out[71] verifies Lemma A.5(6).
(11) Out[74] verifies Lemma A.6(ii).

```
(*We calculate (1920/(x1x2x3x4))f_{a,4,4,3,0} in variables
{a,x1,x2,x3,x4}. This is symmetric in {x1,x2,x3,x4}*)
```

In[1]:=

```
FunctionExpand[1 - (1/8) * x1 * x2 * x3 * x4 * (3 * (5 * a + x1 + x2 + x3 + x4 - 9)/2 - a) * (3 * (5 * a + x1 + x2 +
  x3 + x4 - 9)/2 - 2 * a) * (3 * (5 * a + x1 + x2 + x3 + x4 - 9)/2 - 3 * a) * (3 * (5 * a + x1 + x2 + x3 + x4 - 9)/2 -
  4 * a) + 2 * Binomial[3 * (5 * a + x1 + x2 + x3 + x4 - 9)/2 - 1, 8] - Binomial[x1 - 1, 8] - Binomial[
  x2 - 1, 8] - Binomial[x3 - 1, 8] - Binomial[x4 - 1, 8] - 2 * Binomial[x1 - 1 + 3 * (5 * a + x1 + x2 +
  x3 + x4 - 9)/2, 8] - 2 * Binomial[x2 - 1 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9)/2, 8] - 2 * Binomial[x3 -
  1 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9)/2, 8] - 2 * Binomial[x4 - 1 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9)/2,
  8] + Binomial[x1 + x2 - 1, 8] + Binomial[x1 + x3 - 1, 8] + Binomial[x1 + x4 - 1, 8] + Binomial[
  x2 + x3 - 1, 8] + Binomial[x2 + x4 - 1, 8] + Binomial[x3 + x4 - 1, 8] + 2 * Binomial[x1 + x2 - 1 +
  3 * (5 * a + x1 + x2 + x3 + x4 - 9)/2, 8] + 2 * Binomial[x1 + x3 - 1 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9)/2,
  8] + 2 * Binomial[x1 + x4 - 1 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9)/2, 8] + 2 * Binomial[x2 + x3 - 1 + 3 *
  (5 * a + x1 + x2 + x3 + x4 - 9)/2, 8] + 2 * Binomial[x2 + x4 - 1 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9)/2,
  8] + 2 * Binomial[x3 + x4 - 1 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9)/2, 8] - Binomial[x1 + x2 + x3 - 1,
  8] - Binomial[x1 + x2 + x4 - 1, 8] - Binomial[x1 + x3 + x4 - 1, 8] - Binomial[x2 + x3 + x4 - 1, 8] -
  2 * Binomial[x1 + x2 + x3 - 1 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9)/2, 8] - 2 * Binomial[x1 + x2 + x4 -
  1 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9)/2, 8] - 2 * Binomial[x2 + x3 + x4 - 1 + 3 * (5 * a + x1 + x2 + x3 +
  x4 - 9)/2, 8] - 2 * Binomial[x1 + x3 + x4 - 1 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9)/2, 8] + Binomial[
  x1 + x2 + x3 + x4 - 1, 8] + 2 * Binomial[x1 + x2 + x3 + x4 - 1 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9)/2, 8]]
```

Out[1]=

$$1 - \frac{(-8+x1)(-7+x1)(-6+x1)(-5+x1)(-4+x1)(-3+x1)(-2+x1)(-1+x1)}{40320} - \frac{(-8+x2)(-7+x2)(-6+x2)\cdots 3 \cdots(-2+x2)(-1+x2)}{40320} + \cdots 42 \cdots +$$

$$\frac{(-7+3a+x1+x2+x3+x4)(-43+15a+5x1+5x2+5x3+5x4)\cdots 5 \cdots(-29+15a+5x1+5x2+5x3+5x4)}{1032192} - \frac{1}{8}x1\,x2\,x3\,x4\left(-4a+\frac{3}{2}(-9+5a+x1+x2+x3+x4)\right)$$

$$\left(-3a+\frac{3}{2}(-9+5a+x1+x2+x3+x4)\right)\left(-2a+\frac{3}{2}(-9+5a+x1+x2+x3+x4)\right)\left(-a+\frac{3}{2}(-9+5a+x1+x2+x3+x4)\right)$$

Full expression not available (original memory size: 75.6 kB)

In[2]:=
```
Expand[1920 / (x1 * x2 * x3 * x4) * %1]
```

Out[2]=

$$
\begin{aligned}
&8\,617\,203 - 15\,989\,400\,a + 11\,003\,850\,a^2 - 3\,321\,000\,a^3 + 371\,115\,a^4 - 3\,883\,140\,x1 + 5\,373\,000\,a\,x1 - \\
&2\,454\,300\,a^2\,x1 + 369\,000\,a^3\,x1 + 679\,770\,x1^2 - 621\,000\,a\,x1^2 + 140\,850\,a^2\,x1^2 - 54\,540\,x1^3 + \\
&24\,600\,a\,x1^3 + 1683\,x1^4 - 3\,883\,140\,x2 + 5\,373\,000\,a\,x2 - 2\,454\,300\,a^2\,x2 + 369\,000\,a^3\,x2 + \\
&1\,306\,260\,x1\,x2 - 1\,198\,800\,a\,x1\,x2 + 272\,700\,a^2\,x1\,x2 - 151\,740\,x1^2\,x2 + 69\,000\,a\,x1^2\,x2 + \\
&6060\,x1^3\,x2 + 679\,770\,x2^2 - 621\,000\,a\,x2^2 + 140\,850\,a^2\,x2^2 - 151\,740\,x1\,x2^2 + 69\,000\,a\,x1\,x2^2 + \\
&8770\,x1^2\,x2^2 - 54\,540\,x2^3 + 24\,600\,a\,x2^3 + 6060\,x1\,x2^3 + 1683\,x2^4 - 3\,883\,140\,x3 + 5\,373\,000\,a\,x3 - \\
&2\,454\,300\,a^2\,x3 + 369\,000\,a^3\,x3 + 1\,306\,260\,x1\,x3 - 1\,198\,800\,a\,x1\,x3 + 272\,700\,a^2\,x1\,x3 - \\
&151\,740\,x1^2\,x3 + 69\,000\,a\,x1^2\,x3 + 6060\,x1^3\,x3 + 1\,306\,260\,x2\,x3 - 1\,198\,800\,a\,x2\,x3 + 272\,700\,a^2\,x2\,x3 - \\
&291\,600\,x1\,x2\,x3 + 133\,200\,a\,x1\,x2\,x3 + 16\,860\,x1^2\,x2\,x3 - 151\,740\,x2^2\,x3 + 69\,000\,a\,x2^2\,x3 + \\
&16\,860\,x1\,x2^2\,x3 + 6060\,x2^3\,x3 + 679\,770\,x3^2 - 621\,000\,a\,x3^2 + 140\,850\,a^2\,x3^2 - 151\,740\,x1\,x3^2 + \\
&69\,000\,a\,x1\,x3^2 + 8770\,x1^2\,x3^2 - 151\,740\,x2\,x3^2 + 69\,000\,a\,x2\,x3^2 + 16\,860\,x1\,x2\,x3^2 + 8770\,x2^2\,x3^2 - \\
&54\,540\,x3^3 + 24\,600\,a\,x3^3 + 6060\,x1\,x3^3 + 6060\,x2\,x3^3 + 1683\,x3^4 - 3\,883\,140\,x4 + 5\,373\,000\,a\,x4 - \\
&2\,454\,300\,a^2\,x4 + 369\,000\,a^3\,x4 + 1\,306\,260\,x1\,x4 - 1\,198\,800\,a\,x1\,x4 + 272\,700\,a^2\,x1\,x4 - \\
&151\,740\,x1^2\,x4 + 69\,000\,a\,x1^2\,x4 + 6060\,x1^3\,x4 + 1\,306\,260\,x2\,x4 - 1\,198\,800\,a\,x2\,x4 + \\
&272\,700\,a^2\,x2\,x4 - 291\,600\,x1\,x2\,x4 + 133\,200\,a\,x1\,x2\,x4 + 16\,860\,x1^2\,x2\,x4 - 151\,740\,x2^2\,x4 + \\
&69\,000\,a\,x2^2\,x4 + 16\,860\,x1\,x2^2\,x4 + 6060\,x2^3\,x4 + 1\,306\,260\,x3\,x4 - 1\,198\,800\,a\,x3\,x4 + \\
&272\,700\,a^2\,x3\,x4 - 291\,600\,x1\,x3\,x4 + 133\,200\,a\,x1\,x3\,x4 + 16\,860\,x1^2\,x3\,x4 - 291\,600\,x2\,x3\,x4 + \\
&133\,200\,a\,x2\,x3\,x4 + 32\,400\,x1\,x2\,x3\,x4 + 16\,860\,x2^2\,x3\,x4 - 151\,740\,x3^2\,x4 + 69\,000\,a\,x3^2\,x4 + \\
&16\,860\,x1\,x3^2\,x4 + 16\,860\,x2\,x3^2\,x4 + 6060\,x3^3\,x4 + 679\,770\,x4^2 - 621\,000\,a\,x4^2 + 140\,850\,a^2\,x4^2 - \\
&151\,740\,x1\,x4^2 + 69\,000\,a\,x1\,x4^2 + 8770\,x1^2\,x4^2 - 151\,740\,x2\,x4^2 + 69\,000\,a\,x2\,x4^2 + 16\,860\,x1\,x2\,x4^2 + \\
&8770\,x2^2\,x4^2 - 151\,740\,x3\,x4^2 + 69\,000\,a\,x3\,x4^2 + 16\,860\,x1\,x3\,x4^2 + 16\,860\,x2\,x3\,x4^2 + \\
&8770\,x3^2\,x4^2 - 54\,540\,x4^3 + 24\,600\,a\,x4^3 + 6060\,x1\,x4^3 + 6060\,x2\,x4^3 + 6060\,x3\,x4^3 + 1683\,x4^4
\end{aligned}
$$

```
(*We calculate all the coefficients of the monomial symmetric
polynomials in {x1,x2,x3,x4} appearing in (1920/(x1x2x3x4))f_{a,4,4,3,0}*)
```

In[3]:=
```
a1 = SeriesCoefficient[%2, {x1, 0, 4}, {x2, 0, 0}, {x3, 0, 0}, {x4, 0, 0}]
```

Out[3]= 1683

In[4]:=
```
a2 = SeriesCoefficient[%2, {x1, 0, 3}, {x2, 0, 1}, {x3, 0, 0}, {x4, 0, 0}]
```

Out[4]= 6060

In[5]:=
```
a3 = SeriesCoefficient[%2, {x1, 0, 2}, {x2, 0, 2}, {x3, 0, 0}, {x4, 0, 0}]
```

Out[5]= 8770

In[6]:=
```
a4 = SeriesCoefficient[%2, {x1, 0, 2}, {x2, 0, 1}, {x3, 0, 1}, {x4, 0, 0}]
```

Out[6]= 16 860

In[7]:=
```
a5 = SeriesCoefficient[%2, {x1, 0, 1}, {x2, 0, 1}, {x3, 0, 1}, {x4, 0, 1}]
```

Out[7]= 32 400

In[8]:=
```
l6 = SeriesCoefficient[%2, {x1, 0, 3}, {x2, 0, 0}, {x3, 0, 0}, {x4, 0, 0}]
```

Out[8]= $-54\,540 + 24\,600\,a$

In[9]:= `l7 = SeriesCoefficient[%2, {x1, 0, 2}, {x2, 0, 1}, {x3, 0, 0}, {x4, 0, 0}]`

Out[9]= $-151740 + 69000\, a$

In[10]:= `l8 = SeriesCoefficient[%2, {x1, 0, 1}, {x2, 0, 1}, {x3, 0, 1}, {x4, 0, 0}]`

Out[10]=

$-291600 + 133200\, a$

In[11]:= `l9 = SeriesCoefficient[%2, {x1, 0, 2}, {x2, 0, 0}, {x3, 0, 0}, {x4, 0, 0}]`

Out[11]=

$679770 - 621000\, a + 140850\, a^2$

In[12]:= `l10 = SeriesCoefficient[%2, {x1, 0, 1}, {x2, 0, 1}, {x3, 0, 0}, {x4, 0, 0}]`

Out[12]=

$1306260 - 1198800\, a + 272700\, a^2$

In[13]:= `l11 = SeriesCoefficient[%2, {x1, 0, 1}, {x2, 0, 0}, {x3, 0, 0}, {x4, 0, 0}]`

Out[13]=

$-3883140 + 5373000\, a - 2454300\, a^2 + 369000\, a^3$

In[14]:= `l12 = SeriesCoefficient[%2, {x1, 0, 0}, {x2, 0, 0}, {x3, 0, 0}, {x4, 0, 0}]`

Out[14]=

$8617203 - 15989400\, a + 11003850\, a^2 - 3321000\, a^3 + 371115\, a^4$

```
(*We calculate all the coefficients of the monomial symmetric polynomials in {x_
1,...,x_s} appearing in (1920/(product x_i))f_{a,4,s,3,0} using Lemma A.6 of [LR2]*)
```

In[15]:= `a6 = Expand[l6 - (s - 4) * (a2)]`

Out[15]=

$-30300 + 24600\, a - 6060\, s$

In[16]:= `a7 = Expand[l7 - (s - 4) * (a4)]`

Out[16]=

$-84300 + 69000\, a - 16860\, s$

In[17]:= `a8 = Expand[l8 - (s - 4) * (a5)]`

Out[17]=

$-162000 + 133200\, a - 32400\, s$

In[18]:= `a9 = Expand[l9 - (s - 4) * (a3 + a7) - Binomial[s - 4, 2] * (a4)]`

Out[18]=

$209050 - 345000\, a + 140850\, a^2 + 83960\, s - 69000\, a\, s + 8430\, s^2$

In[19]:= `a10 = Expand[l10 - (s - 4) * (a4 + a8) - Binomial[s - 4, 2] * (a5)]`

Out[19]=

$401700 - 666000\, a + 272700\, a^2 + 161340\, s - 133200\, a\, s + 16200\, s^2$

In[20]:=
```
a11 = Expand[l11 - (s - 4) * (a2 + a7 + a10) - Binomial[s - 4, 2] * (2 * a4 + a8) - Binomial[s - 4, 3] * (a5)]
```

Out[20]=

$$-658\,500 + 1\,653\,000\,a - 1\,363\,500\,a^2 + 369\,000\,a^3 - 398\,400\,s + 663\,600\,a\,s - 272\,700\,a^2\,s - 80\,340\,s^2 + 66\,600\,a\,s^2 - 5400\,s^3$$

In[21]:=
```
a12 = Expand[l12 - (s - 4) * (a1 + a6 + a9 + a11) - Binomial[s - 4, 2] * (2 *
  a2 + a3 + 2 * a7 + a10) - Binomial[s - 4, 3] * (3 * a4 + a8) - Binomial[s - 4, 4] * (a5)]
```

Out[21]=

$$802\,635 - 2\,715\,000\,a + 3\,386\,250\,a^2 - 1\,845\,000\,a^3 + 371\,115\,a^4 + 650\,302\,s - 1\,641\,000\,a\,s + 1\,359\,000\,a^2\,s - 369\,000\,a^3\,s + 197\,555\,s^2 - 330\,600\,a\,s^2 + 136\,350\,a^2\,s^2 + 26\,670\,s^3 - 22\,200\,a\,s^3 + 1350\,s^4$$

```
(*We calculate (1920/(x1x2x3x4))f_{a,4,4,3,1} in variables
{a,x1,x2,x3,x4}. This is symmetric in {x1,x2,x3,x4}*)
```

In[22]:=
```
FunctionExpand[9 - (1 / 8) * x1 * x2 * x3 * x4 * (3 * (5 * a + x1 + x2 + x3 + x4 - 9) / 2 - 1 - a) * (3 * (5 * a + x1 +
  x2 + x3 + x4 - 9) / 2 - 1 - 2 * a) * (3 * (5 * a + x1 + x2 + x3 + x4 - 9) / 2 - 1 - 3 * a) * (3 * (5 * a + x1 + x2 + x3 +
  x4 - 9) / 2 - 1 - 4 * a) + 2 * Binomial[3 * (5 * a + x1 + x2 + x3 + x4 - 9) / 2 - 2, 8] - Binomial[x1 - 2,
  8] - Binomial[x2 - 2, 8] - Binomial[x3 - 2, 8] - Binomial[x4 - 2, 8] - 2 * Binomial[x1 - 2 + 3 *
  (5 * a + x1 + x2 + x3 + x4 - 9) / 2, 8] - 2 * Binomial[x2 - 2 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9) / 2, 8] - 2 *
  Binomial[x3 - 2 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9) / 2, 8] - 2 * Binomial[x4 - 2 + 3 * (5 * a + x1 + x2 +
  x3 + x4 - 9) / 2, 8] + Binomial[x1 + x2 - 2, 8] + Binomial[x1 + x3 - 2, 8] + Binomial[x1 + x4 - 2, 8] +
  Binomial[x2 + x3 - 2, 8] + Binomial[x2 + x4 - 2, 8] + Binomial[x3 + x4 - 2, 8] + 2 * Binomial[x1 +
  x2 - 2 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9) / 2, 8] + 2 * Binomial[x1 + x3 - 2 + 3 * (5 * a + x1 + x2 + x3 + x4 -
  9) / 2, 8] + 2 * Binomial[x1 + x4 - 2 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9) / 2, 8] + 2 * Binomial[x2 + x3 -
  2 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9) / 2, 8] + 2 * Binomial[x2 + x4 - 2 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9) /
  2, 8] + 2 * Binomial[x3 + x4 - 2 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9) / 2, 8] - Binomial[x1 + x2 + x3 - 2,
  8] - Binomial[x1 + x2 + x4 - 2, 8] - Binomial[x1 + x3 + x4 - 2, 8] - Binomial[x2 + x3 + x4 - 2, 8] -
  2 * Binomial[x1 + x2 + x3 - 2 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9) / 2, 8] - 2 * Binomial[x1 + x2 + x4 -
  2 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9) / 2, 8] - 2 * Binomial[x2 + x3 + x4 - 2 + 3 * (5 * a + x1 + x2 + x3 +
  x4 - 9) / 2, 8] - 2 * Binomial[x1 + x3 + x4 - 2 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9) / 2, 8] + Binomial[
  x1 + x2 + x3 + x4 - 2, 8] + 2 * Binomial[x1 + x2 + x3 + x4 - 2 + 3 * (5 * a + x1 + x2 + x3 + x4 - 9) / 2, 8]]
```

Out[22]=

$$9 - \frac{(-9+x1)\,(-8+x1)\,(-7+x1)\,(-6+x1)\,(-5+x1)\,(-4+x1)\,(-3+x1)\,(-2+x1)}{40\,320} - \frac{(-9+x2)\,(-8+x2)\,\cdots 4 \cdots\,(-3+x2)\,(-2+x2)}{40\,320} + \cdots 42 \cdots +$$
$$\frac{5\,(-9+3\,a+x1+x2+x3+x4)\,\cdots 5 \cdots\,(-33+15\,a+5\,x1+5\,x2+5\,x3+5\,x4)\,(-31+15\,a+5\,x1+5\,x2+5\,x3+5\,x4)}{1\,032\,192} - \frac{1}{8}\,x1\,x2\,x3\,x4\left(-1-4\,a+\frac{3}{2}\left(-9+5\,a+x1+x2+x3+x4\right)\right)$$
$$\left(-1-3\,a+\frac{3}{2}\left(-9+5\,a+x1+x2+x3+x4\right)\right)\left(-1-2\,a+\frac{3}{2}\left(-9+5\,a+x1+x2+x3+x4\right)\right)\left(-1-a+\frac{3}{2}\left(-9+5\,a+x1+x2+x3+x4\right)\right)$$

Full expression not available (original memory size: 75.1 kB)

In[23]:=
```
Expand[1920 / (x1 * x2 * x3 * x4) * %22]
```

Out[23]=

$$10\,044\,963 - 18\,084\,600\, a + 12\,010\,650\, a^2 - 3\,477\,000\, a^3 + 371\,115\, a^4 - 4\,359\,420\, x1 + 5\,833\,800\, a\, x1 - 2\,564\,100\, a^2\, x1 + 369\,000\, a^3\, x1 + 735\,690\, x1^2 - 647\,400\, a\, x1^2 + 140\,850\, a^2\, x1^2 - 56\,820\, x1^3 + 24\,600\, a\, x1^3 + 1683\, x1^4 - 4\,359\,420\, x2 + 5\,833\,800\, a\, x2 - 2\,564\,100\, a^2\, x2 + 369\,000\, a^3\, x2 + 1\,411\,380\, x1\, x2 - 1\,249\,200\, a\, x1\, x2 + 272\,700\, a^2\, x1\, x2 - 157\,860\, x1^2\, x2 + 69\,000\, a\, x1^2\, x2 + 6060\, x1^3\, x2 + 735\,690\, x2^2 - 647\,400\, a\, x2^2 + 140\,850\, a^2\, x2^2 - 157\,860\, x1\, x2^2 + 69\,000\, a\, x1\, x2^2 + 8770\, x1^2\, x2^2 - 56\,820\, x2^3 + 24\,600\, a\, x2^3 + 6060\, x1\, x2^3 + 1683\, x2^4 - 4\,359\,420\, x3 + 5\,833\,800\, a\, x3 - 2\,564\,100\, a^2\, x3 + 369\,000\, a^3\, x3 + 1\,411\,380\, x1\, x3 - 1\,249\,200\, a\, x1\, x3 + 272\,700\, a^2\, x1\, x3 - 157\,860\, x1^2\, x3 + 69\,000\, a\, x1^2\, x3 + 6060\, x1^3\, x3 + 1\,411\,380\, x2\, x3 - 1\,249\,200\, a\, x2\, x3 + 272\,700\, a^2\, x2\, x3 - 303\,120\, x1\, x2\, x3 + 133\,200\, a\, x1\, x2\, x3 + 16\,860\, x1^2\, x2\, x3 - 157\,860\, x2^2\, x3 + 69\,000\, a\, x2^2\, x3 + 16\,860\, x1\, x2^2\, x3 + 6060\, x2^3\, x3 + 735\,690\, x3^2 - 647\,400\, a\, x3^2 + 140\,850\, a^2\, x3^2 - 157\,860\, x1\, x3^2 + 69\,000\, a\, x1\, x3^2 + 8770\, x1^2\, x3^2 - 157\,860\, x2\, x3^2 + 69\,000\, a\, x2\, x3^2 + 16\,860\, x1\, x2\, x3^2 + 8770\, x2^2\, x3^2 - 56\,820\, x3^3 + 24\,600\, a\, x3^3 + 6060\, x1\, x3^3 + 6060\, x2\, x3^3 + 1683\, x3^4 - 4\,359\,420\, x4 + 5\,833\,800\, a\, x4 - 2\,564\,100\, a^2\, x4 + 369\,000\, a^3\, x4 + 1\,411\,380\, x1\, x4 - 1\,249\,200\, a\, x1\, x4 + 272\,700\, a^2\, x1\, x4 - 157\,860\, x1^2\, x4 + 69\,000\, a\, x1^2\, x4 + 6060\, x1^3\, x4 + 1\,411\,380\, x2\, x4 - 1\,249\,200\, a\, x2\, x4 + 272\,700\, a^2\, x2\, x4 - 303\,120\, x1\, x2\, x4 + 133\,200\, a\, x1\, x2\, x4 + 16\,860\, x1^2\, x2\, x4 - 157\,860\, x2^2\, x4 + 69\,000\, a\, x2^2\, x4 + 16\,860\, x1\, x2^2\, x4 + 6060\, x2^3\, x4 + 1\,411\,380\, x3\, x4 - 1\,249\,200\, a\, x3\, x4 + 272\,700\, a^2\, x3\, x4 - 303\,120\, x1\, x3\, x4 + 133\,200\, a\, x1\, x3\, x4 + 16\,860\, x1^2\, x3\, x4 - 303\,120\, x2\, x3\, x4 + 133\,200\, a\, x2\, x3\, x4 + 32\,400\, x1\, x2\, x3\, x4 + 16\,860\, x2^2\, x3\, x4 - 157\,860\, x3^2\, x4 + 69\,000\, a\, x3^2\, x4 + 16\,860\, x1\, x3^2\, x4 + 16\,860\, x2\, x3^2\, x4 + 6060\, x3^3\, x4 + 735\,690\, x4^2 - 647\,400\, a\, x4^2 + 140\,850\, a^2\, x4^2 - 157\,860\, x1\, x4^2 + 69\,000\, a\, x1\, x4^2 + 8770\, x1^2\, x4^2 - 157\,860\, x2\, x4^2 + 69\,000\, a\, x2\, x4^2 + 16\,860\, x1\, x2\, x4^2 + 8770\, x2^2\, x4^2 - 157\,860\, x3\, x4^2 + 69\,000\, a\, x3\, x4^2 + 16\,860\, x1\, x3\, x4^2 + 16\,860\, x2\, x3\, x4^2 + 8770\, x3^2\, x4^2 - 56\,820\, x4^3 + 24\,600\, a\, x4^3 + 6060\, x1\, x4^3 + 6060\, x2\, x4^3 + 6060\, x3\, x4^3 + 1683\, x4^4$$

```
(*We calculate all the coefficients of the monomial symmetric
polynomials in {a,b,c,d} appearing in (1920/(x1x2x3x4))f_{a,4,4,3,1}*)
```

In[24]:=
```
b1 = SeriesCoefficient[%23, {x1, 0, 4}, {x2, 0, 0}, {x3, 0, 0}, {x4, 0, 0}]
```

Out[24]=

1683

In[25]:=
```
b2 = SeriesCoefficient[%23, {x1, 0, 3}, {x2, 0, 1}, {x3, 0, 0}, {x4, 0, 0}]
```

Out[25]=

6060

In[26]:=
```
b3 = SeriesCoefficient[%23, {x1, 0, 2}, {x2, 0, 2}, {x3, 0, 0}, {x4, 0, 0}]
```

Out[26]=

8770

In[27]:=
```
b4 = SeriesCoefficient[%23, {x1, 0, 2}, {x2, 0, 1}, {x3, 0, 1}, {x4, 0, 0}]
```

Out[27]=

16 860

In[28]:=
```
b5 = SeriesCoefficient[%23, {x1, 0, 1}, {x2, 0, 1}, {x3, 0, 1}, {x4, 0, 1}]
```

Out[28]=

32 400

```
In[29]:= n6 = SeriesCoefficient[%23, {x1, 0, 3}, {x2, 0, 0}, {x3, 0, 0}, {x4, 0, 0}]
Out[29]=
```
$-56\,820 + 24\,600\,a$

```
In[30]:= n7 = SeriesCoefficient[%23, {x1, 0, 2}, {x2, 0, 1}, {x3, 0, 0}, {x4, 0, 0}]
Out[30]=
```
$-157\,860 + 69\,000\,a$

```
In[31]:= n8 = SeriesCoefficient[%23, {x1, 0, 1}, {x2, 0, 1}, {x3, 0, 1}, {x4, 0, 0}]
Out[31]=
```
$-303\,120 + 133\,200\,a$

```
In[32]:= n9 = SeriesCoefficient[%23, {x1, 0, 2}, {x2, 0, 0}, {x3, 0, 0}, {x4, 0, 0}]
Out[32]=
```
$735\,690 - 647\,400\,a + 140\,850\,a^2$

```
In[33]:= n10 = SeriesCoefficient[%23, {x1, 0, 1}, {x2, 0, 1}, {x3, 0, 0}, {x4, 0, 0}]
Out[33]=
```
$1\,411\,380 - 1\,249\,200\,a + 272\,700\,a^2$

```
In[34]:= n11 = SeriesCoefficient[%23, {x1, 0, 1}, {x2, 0, 0}, {x3, 0, 0}, {x4, 0, 0}]
Out[34]=
```
$-4\,359\,420 + 5\,833\,800\,a - 2\,564\,100\,a^2 + 369\,000\,a^3$

```
In[35]:= n12 = SeriesCoefficient[%23, {x1, 0, 0}, {x2, 0, 0}, {x3, 0, 0}, {x4, 0, 0}]
Out[35]=
```
$10\,044\,963 - 18\,084\,600\,a + 12\,010\,650\,a^2 - 3\,477\,000\,a^3 + 371\,115\,a^4$

```
(*We calculate all the coefficients of the monomial symmetric polynomials in {x_
1,...,x_s} appearing in (1920/(product x_i))f_{a,4,s,3,1} using Lemma A.6 of [LR2]*)

In[36]:= b6 = Expand[n6 - (s - 4) * (b2)]
Out[36]=
```
$-32\,580 + 24\,600\,a - 6060\,s$

```
In[37]:= b7 = Expand[n7 - (s - 4) * (b4)]
Out[37]=
```
$-90\,420 + 69\,000\,a - 16\,860\,s$

```
In[38]:= b8 = Expand[n8 - (s - 4) * (b5)]
Out[38]=
```
$-173\,520 + 133\,200\,a - 32\,400\,s$

```
In[39]:= b9 = Expand[n9 - (s - 4) * (b3 + b7) - Binomial[s - 4, 2] * (b4)]
Out[39]=
```
$240\,490 - 371\,400\,a + 140\,850\,a^2 + 90\,080\,s - 69\,000\,a\,s + 8430\,s^2$

```
In[40]:= b10 = Expand[n10 - (s - 4) * (b4 + b8) - Binomial[s - 4, 2] * (b5)]
Out[40]=
```
$460\,740 - 716\,400\,a + 272\,700\,a^2 + 172\,860\,s - 133\,200\,a\,s + 16\,200\,s^2$

In[41]:=
```
b11 = Expand[n11 - (s - 4) * (b2 + b7 + b10) - Binomial[s - 4, 2] * (2 * b4 + b8) - Binomial[s - 4, 3] * (b5)]
```

Out[41]=

$$-807\,900 + 1\,912\,200\,a - 1\,473\,300\,a^2 + 369\,000\,a^3 - 457\,080\,s + 714\,000\,a\,s - 272\,700\,a^2\,s - 86\,100\,s^2 + 66\,600\,a\,s^2 - 5400\,s^3$$

In[42]:=
```
b12 = Expand[n12 - (s - 4) * (b1 + b6 + b9 + b11) - Binomial[s - 4, 2] * (2 *
  b2 + b3 + 2 * b7 + b10) - Binomial[s - 4, 3] * (3 * b4 + b8) - Binomial[s - 4, 4] * (b5)]
```

Out[42]=

$$1\,051\,035 - 3\,375\,000\,a + 3\,953\,850\,a^2 - 2\,001\,000\,a^3 + 371\,115\,a^4 + 797\,782\,s - 1\,899\,000\,a\,s + 1\,468\,800\,a^2\,s - 369\,000\,a^3\,s + 226\,715\,s^2 - 355\,800\,a\,s^2 + 136\,350\,a^2\,s^2 + 28\,590\,s^3 - 22\,200\,a\,s^3 + 1350\,s^4$$

```
(*Defining the functions computing (1/(product x_i))
f_{a,4,s,3,0} and (1/(product x_i))f_{a,4,s,3,1}*)
```

In[43]:=
```
f0 = a1 * m4 + a2 * m31 + a3 * m22 + a4 * m211 + a5 * m1111 +
  a6 * m3 + a7 * m21 + a8 * m111 + a9 * m2 + a10 * m11 + a11 * m1 + a12
```

Out[43]=

$$802\,635 - 2\,715\,000\,a + 3\,386\,250\,a^2 - 1\,845\,000\,a^3 + 371\,115\,a^4 + 32\,400\,\mathrm{m1111} + 16\,860\,\mathrm{m211} + 8770\,\mathrm{m22} + 6060\,\mathrm{m31} + 1683\,\mathrm{m4} + \mathrm{m111}\,(-162\,000 + 133\,200\,a - 32\,400\,s) + \mathrm{m21}\,(-84\,300 + 69\,000\,a - 16\,860\,s) + \mathrm{m3}\,(-30\,300 + 24\,600\,a - 6060\,s) + 650\,302\,s - 1\,641\,000\,a\,s + 1\,359\,000\,a^2\,s - 369\,000\,a^3\,s + 197\,555\,s^2 - 330\,600\,a\,s^2 + 136\,350\,a^2\,s^2 + 26\,670\,s^3 - 22\,200\,a\,s^3 + 1350\,s^4 + \mathrm{m2}\,(209\,050 - 345\,000\,a + 140\,850\,a^2 + 83\,960\,s - 69\,000\,a\,s + 8430\,s^2) + \mathrm{m11}\,(401\,700 - 666\,000\,a + 272\,700\,a^2 + 161\,340\,s - 133\,200\,a\,s + 16\,200\,s^2) + \mathrm{m1}\,(-658\,500 + 1\,653\,000\,a - 1\,363\,500\,a^2 + 369\,000\,a^3 - 398\,400\,s + 663\,600\,a\,s - 272\,700\,a^2\,s - 80\,340\,s^2 + 66\,600\,a\,s^2 - 5400\,s^3)$$

In[44]:=
```
f1 = b1 * m4 + b2 * m31 + b3 * m22 + b4 * m211 + b5 * m1111 +
  b6 * m3 + b7 * m21 + b8 * m111 + b9 * m2 + b10 * m11 + b11 * m1 + b12
```

Out[44]=

$$1\,051\,035 - 3\,375\,000\,a + 3\,953\,850\,a^2 - 2\,001\,000\,a^3 + 371\,115\,a^4 + 32\,400\,\mathrm{m1111} + 16\,860\,\mathrm{m211} + 8770\,\mathrm{m22} + 6060\,\mathrm{m31} + 1683\,\mathrm{m4} + \mathrm{m111}\,(-173\,520 + 133\,200\,a - 32\,400\,s) + \mathrm{m21}\,(-90\,420 + 69\,000\,a - 16\,860\,s) + \mathrm{m3}\,(-32\,580 + 24\,600\,a - 6060\,s) + 797\,782\,s - 1\,899\,000\,a\,s + 1\,468\,800\,a^2\,s - 369\,000\,a^3\,s + 226\,715\,s^2 - 355\,800\,a\,s^2 + 136\,350\,a^2\,s^2 + 28\,590\,s^3 - 22\,200\,a\,s^3 + 1350\,s^4 + \mathrm{m2}\,(240\,490 - 371\,400\,a + 140\,850\,a^2 + 90\,080\,s - 69\,000\,a\,s + 8430\,s^2) + \mathrm{m11}\,(460\,740 - 716\,400\,a + 272\,700\,a^2 + 172\,860\,s - 133\,200\,a\,s + 16\,200\,s^2) + \mathrm{m1}\,(-807\,900 + 1\,912\,200\,a - 1\,473\,300\,a^2 + 369\,000\,a^3 - 457\,080\,s + 714\,000\,a\,s - 272\,700\,a^2\,s - 86\,100\,s^2 + 66\,600\,a\,s^2 - 5400\,s^3)$$

In[45]:= **Fa4s30 = (1 / 1920) * f0**

Out[45]=

$$\frac{1}{1920}(802\,635 - 2\,715\,000\,a + 3\,386\,250\,a^2 - 1\,845\,000\,a^3 + 371\,115\,a^4 + 32\,400\,m1111 + 16\,860\,m211 + 8770\,m22 + 6060\,m31 + 1683\,m4 + m111\,(-162\,000 + 133\,200\,a - 32\,400\,s) + m21\,(-84\,300 + 69\,000\,a - 16\,860\,s) + m3\,(-30\,300 + 24\,600\,a - 6060\,s) + 650\,302\,s - 1\,641\,000\,a\,s + 1\,359\,000\,a^2\,s - 369\,000\,a^3\,s + 197\,555\,s^2 - 330\,600\,a\,s^2 + 136\,350\,a^2\,s^2 + 26\,670\,s^3 - 22\,200\,a\,s^3 + 1350\,s^4 + m2\,(209\,050 - 345\,000\,a + 140\,850\,a^2 + 83\,960\,s - 69\,000\,a\,s + 8430\,s^2) + m11\,(401\,700 - 666\,000\,a + 272\,700\,a^2 + 161\,340\,s - 133\,200\,a\,s + 16\,200\,s^2) + m1\,(-658\,500 + 1\,653\,000\,a - 1\,363\,500\,a^2 + 369\,000\,a^3 - 398\,400\,s + 663\,600\,a\,s - 272\,700\,a^2\,s - 80\,340\,s^2 + 66\,600\,a\,s^2 - 5400\,s^3))$$

In[46]:= **Fa4s31 = (1 / 1920) * f1**

Out[46]=

$$\frac{1}{1920}(1\,051\,035 - 3\,375\,000\,a + 3\,953\,850\,a^2 - 2\,001\,000\,a^3 + 371\,115\,a^4 + 32\,400\,m1111 + 16\,860\,m211 + 8770\,m22 + 6060\,m31 + 1683\,m4 + m111\,(-173\,520 + 133\,200\,a - 32\,400\,s) + m21\,(-90\,420 + 69\,000\,a - 16\,860\,s) + m3\,(-32\,580 + 24\,600\,a - 6060\,s) + 797\,782\,s - 1\,899\,000\,a\,s + 1\,468\,800\,a^2\,s - 369\,000\,a^3\,s + 226\,715\,s^2 - 355\,800\,a\,s^2 + 136\,350\,a^2\,s^2 + 28\,590\,s^3 - 22\,200\,a\,s^3 + 1350\,s^4 + m2\,(240\,490 - 371\,400\,a + 140\,850\,a^2 + 90\,080\,s - 69\,000\,a\,s + 8430\,s^2) + m11\,(460\,740 - 716\,400\,a + 272\,700\,a^2 + 172\,860\,s - 133\,200\,a\,s + 16\,200\,s^2) + m1\,(-807\,900 + 1\,912\,200\,a - 1\,473\,300\,a^2 + 369\,000\,a^3 - 457\,080\,s + 714\,000\,a\,s - 272\,700\,a^2\,s - 86\,100\,s^2 + 66\,600\,a\,s^2 - 5400\,s^3))$$

(*We compute the polynomial calculating (24/(rd))deg(Z) Lemma 4.1(iv) where X is in P^{m+s}, c.i. of type (d_1,...,d_s), E Ulrich for (X,O_X(a)) using Lemma 2.2(ii)*)

In[47]:= **Expand[(24 / (r * d)) ((1 / 2) * ((r / 2) * ((m + 1) * (a - 1) + m1 - s)) ^ 2 * d - (1 / 2) * ((r / 2) * ((m + 1) * (a - 1) + m1 - s)) * (m1 - s - m - 1) * d + (r / 12) * (m1 - s - m - 1) ^ 2 * d + (r * d / 12) * (Binomial[m + s + 1, 2] + m1 * (m1 - s - m - 1) - m11) - (r * d / 24) * a ^ 2 * (3 * m ^ 2 + 5 * m + 2))]**

Out[47]=

$$-4 + 6\,a - 2\,a^2 - 7\,m + 12\,a\,m - 5\,a^2\,m - 3\,m^2 + 6\,a\,m^2 - 3\,a^2\,m^2 + 6\,m1 - 6\,a\,m1 + 6\,m\,m1 - 6\,a\,m\,m1 - 2\,m1^2 - 2\,m11 + 3\,r - 6\,a\,r + 3\,a^2\,r + 6\,m\,r - 12\,a\,m\,r + 6\,a^2\,m\,r + 3\,m^2\,r - 6\,a\,m^2\,r + 3\,a^2\,m^2\,r - 6\,m1\,r + 6\,a\,m1\,r - 6\,m\,m1\,r + 6\,a\,m\,m1\,r + 3\,m1^2\,r - 7\,s + 6\,a\,s - 6\,m\,s + 6\,a\,m\,s + 6\,m1\,s + 6\,r\,s - 6\,a\,r\,s + 6\,m\,r\,s - 6\,a\,m\,r\,s - 6\,m1\,r\,s - 3\,s^2 + 3\,r\,s^2$$

(*Specializing the above when r=3,m=4*)

In[48]:= **dul = %47 /. {r → 3, m → 4}**

Out[48]=

$$145 - 300\,a + 155\,a^2 - 60\,m1 + 60\,a\,m1 + 7\,m1^2 - 2\,m11 + 59\,s - 60\,a\,s - 12\,m1\,s + 6\,s^2$$

In[49]:= **%48 /. {(m1) ^ 2 → m2 + 2 * m11}**

Out[49]=

$$145 - 300\,a + 155\,a^2 - 60\,m1 + 60\,a\,m1 - 2\,m11 + 7\,(2\,m11 + m2) + 59\,s - 60\,a\,s - 12\,m1\,s + 6\,s^2$$

```
In[50]:= d1 = Expand[%49]
Out[50]=
145 - 300 a + 155 a^2 - 60 m1 + 60 a m1 + 12 m11 + 7 m2 + 59 s - 60 a s - 12 m1 s + 6 s^2

(*Polynomial calculating (8/d)H_ZK_Z*)

In[51]:= d2 = Expand[(1 / 120) * (f0 - f1) + d1]
Out[51]=
-1925 + 5200 a - 4575 a^2 + 1300 a^3 + 1185 m1 - 2100 a m1 + 915 a^2 m1 - 480 m11 +
 420 a m11 + 96 m111 - 255 m2 + 220 a m2 + 51 m21 + 19 m3 - 1170 s + 2090 a s - 915 a^2 s +
 477 m1 s - 420 a m1 s - 96 m11 s - 51 m2 s - 237 s^2 + 210 a s^2 + 48 m1 s^2 - 16 s^3

(*Calculations of (32/(5d))K_Z^2 using Remark 4.4(ix) of [LR2]*)

In[52]:= Expand[2 * ((5 / 2) * (m1 - s) + (3 / 2) * (5 * a - 5) - 5) * d2]
Out[52]=
48 125 - 158 875 a + 192 375 a^2 - 101 125 a^3 + 19 500 a^4 - 39 250 m1 + 96 275 a m1 - 77 250 a^2 m1 +
 20 225 a^3 m1 + 5925 m1^2 - 10 500 a m1^2 + 4575 a^2 m1^2 + 12 000 m11 - 17 700 a m11 + 6300 a^2 m11 -
 2400 m1 m11 + 2100 a m1 m11 - 2400 m111 + 1440 a m111 + 480 m1 m111 + 6375 m2 - 9325 a m2 +
 3300 a^2 m2 - 1275 m1 m2 + 1100 a m1 m2 - 1275 m21 + 765 a m21 + 255 m1 m21 - 475 m3 + 285 a m3 +
 95 m1 m3 + 38 875 s - 95 800 a s + 77 100 a^2 s - 20 225 a^3 s - 23 700 m1 s + 38 605 a m1 s - 15 450 a^2 m1 s +
 2385 m1^2 s - 2100 a m1^2 s + 4800 m11 s - 3540 a m11 s - 480 m1 m11 s - 480 m111 s + 2550 m2 s -
 1865 a m2 s - 255 m1 m2 s - 255 m21 s - 95 m3 s + 11 775 s^2 - 19 255 a s^2 + 7725 a^2 s^2 - 4770 m1 s^2 +
 3870 a m1 s^2 + 240 m1^2 s^2 + 480 m11 s^2 + 255 m2 s^2 + 1585 s^3 - 1290 a s^3 - 320 m1 s^3 + 80 s^4

In[53]:= %52 /. {(m1)^2 → m2 + 2 * m11, (m1)^3 → m3 + 3 * m21 + 6 * m111, (m1)^4 → m4 + 4 * m31 +
 6 * m22 + 12 * m211 + 24 * m1111, m1 * m11 → m21 + 3 * m111, (m1)^2 * m11 → m31 + 2 * m22 +
 5 * m211 + 12 * m1111, (m11)^2 → m22 + 2 * m211 + 6 * m1111, m1 * m3 → m4 + m31, m1 *
 m21 → m31 + 2 * m22 + 2 * m211, m1 * m111 → m211 + 4 * m1111, m1 * m2 → m3 + m21, (m1)^
 2 * m2 → m4 + 2 * m31 + 2 * m22 + 2 * m211, (m2)^2 → m4 + 2 * m22, m2 * m11 → m31 + m211}
Out[53]=
48 125 - 158 875 a + 192 375 a^2 - 101 125 a^3 + 19 500 a^4 - 39 250 m1 + 96 275 a m1 - 77 250 a^2 m1 +
 20 225 a^3 m1 + 12 000 m11 - 17 700 a m11 + 6300 a^2 m11 - 2400 m111 + 1440 a m111 + 6375 m2 -
 9325 a m2 + 3300 a^2 m2 + 5925 (2 m11 + m2) - 10 500 a (2 m11 + m2) + 4575 a^2 (2 m11 + m2) - 1275 m21 +
 765 a m21 - 2400 (3 m111 + m21) + 2100 a (3 m111 + m21) + 480 (4 m1111 + m211) - 475 m3 + 285 a m3 -
 1275 (m21 + m3) + 1100 a (m21 + m3) + 255 (2 m211 + 2 m22 + m31) + 95 (m31 + m4) + 38 875 s - 95 800 a s +
 77 100 a^2 s - 20 225 a^3 s - 23 700 m1 s + 38 605 a m1 s - 15 450 a^2 m1 s + 4800 m11 s - 3540 a m11 s -
 480 m111 s + 2550 m2 s - 1865 a m2 s + 2385 (2 m11 + m2) s - 2100 a (2 m11 + m2) s - 255 m21 s -
 480 (3 m111 + m21) s - 95 m3 s - 255 (m21 + m3) s + 11 775 s^2 - 19 255 a s^2 + 7725 a^2 s^2 - 4770 m1 s^2 +
 3870 a m1 s^2 + 480 m11 s^2 + 255 m2 s^2 + 240 (2 m11 + m2) s^2 + 1585 s^3 - 1290 a s^3 - 320 m1 s^3 + 80 s^4
```

```
In[54]:= d3 = Expand[%53]

Out[54]=
48 125 - 158 875 a + 192 375 a^2 - 101 125 a^3 + 19 500 a^4 - 39 250 m1 + 96 275 a m1 - 77 250 a^2 m1 +
  20 225 a^3 m1 + 23 850 m11 - 38 700 a m11 + 15 450 a^2 m11 - 9600 m111 + 7740 a m111 +
  1920 m1111 + 12 300 m2 - 19 825 a m2 + 7875 a^2 m2 - 4950 m21 + 3965 a m21 + 990 m211 + 510 m22 -
  1750 m3 + 1385 a m3 + 350 m31 + 95 m4 + 38 875 s - 95 800 a s + 77 100 a^2 s - 20 225 a^3 s -
  23 700 m1 s + 38 605 a m1 s - 15 450 a^2 m1 s + 9570 m11 s - 7740 a m11 s - 1920 m111 s +
  4935 m2 s - 3965 a m2 s - 990 m21 s - 350 m3 s + 11 775 s^2 - 19 255 a s^2 + 7725 a^2 s^2 -
  4770 m1 s^2 + 3870 a m1 s^2 + 960 m11 s^2 + 495 m2 s^2 + 1585 s^3 - 1290 a s^3 - 320 m1 s^3 + 80 s^4

In[55]:= Expand[4 * ((5 / 2) * (m1 - s) + (3 / 2) * (5 * a - 5) - 5) ^ 2]

Out[55]=
625 - 750 a + 225 a^2 - 250 m1 + 150 a m1 + 25 m1^2 + 250 s - 150 a s - 50 m1 s + 25 s^2

In[56]:= Expand[%55 /. {(m1) ^ 2 → m2 + 2 * m11}]

Out[56]=
625 - 750 a + 225 a^2 - 250 m1 + 150 a m1 + 50 m11 + 25 m2 + 250 s - 150 a s - 50 m1 s + 25 s^2

In[57]:= Expand[%56 * d1]

Out[57]=
90 625 - 296 250 a + 354 500 a^2 - 183 750 a^3 + 34 875 a^4 - 73 750 m1 + 179 250 a m1 - 142 250 a^2 m1 +
  36 750 a^3 m1 + 15 000 m1^2 - 24 000 a m1^2 + 9000 a^2 m1^2 + 14 750 m11 - 24 000 a m11 + 10 450 a^2 m11 -
  6000 m1 m11 + 4800 a m1 m11 + 600 m11^2 + 8000 m2 - 12 750 a m2 + 5450 a^2 m2 - 3250 m1 m2 +
  2550 a m1 m2 + 650 m11 m2 + 175 m2^2 + 73 125 s - 178 500 a s + 142 025 a^2 s - 36 750 a^3 s - 44 500 m1 s +
  71 850 a m1 s - 28 450 a^2 m1 s + 6000 m1^2 s - 4800 a m1^2 s + 5950 m11 s - 4800 a m11 s - 1200 m1 m11 s +
  3225 m2 s - 2550 a m2 s - 650 m1 m2 s + 22 125 s^2 - 35 850 a s^2 + 14 225 a^2 s^2 - 8950 m1 s^2 +
  7200 a m1 s^2 + 600 m1^2 s^2 + 600 m11 s^2 + 325 m2 s^2 + 2975 s^3 - 2400 a s^3 - 600 m1 s^3 + 150 s^4

In[58]:= %57 /. {(m1) ^ 2 → m2 + 2 * m11, (m1) ^ 3 → m3 + 3 * m21 + 6 * m111, (m1) ^ 4 → m4 + 4 * m31 +
  6 * m22 + 12 * m211 + 24 * m1111, m1 * m11 → m21 + 3 * m111, (m1) ^ 2 * m11 → m31 + 2 * m22 +
  5 * m211 + 12 * m1111, (m11) ^ 2 → m22 + 2 * m211 + 6 * m1111, m1 * m3 → m4 + m31, m1 *
  m21 → m31 + 2 * m22 + 2 * m211, m1 * m111 → m211 + 4 * m1111, m1 * m2 → m3 + m21, (m1) ^
  2 * m2 → m4 + 2 * m31 + 2 * m22 + 2 * m211, (m2) ^ 2 → m4 + 2 * m22, m2 * m11 → m31 + m211}

Out[58]=
90 625 - 296 250 a + 354 500 a^2 - 183 750 a^3 + 34 875 a^4 - 73 750 m1 + 179 250 a m1 -
  142 250 a^2 m1 + 36 750 a^3 m1 + 14 750 m11 - 24 000 a m11 + 10 450 a^2 m11 + 8000 m2 -
  12 750 a m2 + 5450 a^2 m2 + 15 000 (2 m11 + m2) - 24 000 a (2 m11 + m2) + 9000 a^2 (2 m11 + m2) -
  6000 (3 m111 + m21) + 4800 a (3 m111 + m21) + 600 (6 m1111 + 2 m211 + m22) - 3250 (m21 + m3) +
  2550 a (m21 + m3) + 650 (m211 + m31) + 175 (2 m22 + m4) + 73 125 s - 178 500 a s + 142 025 a^2 s -
  36 750 a^3 s - 44 500 m1 s + 71 850 a m1 s - 28 450 a^2 m1 s + 5950 m11 s - 4800 a m11 s +
  3225 m2 s - 2550 a m2 s + 6000 (2 m11 + m2) s - 4800 a (2 m11 + m2) s - 1200 (3 m111 + m21) s -
  650 (m21 + m3) s + 22 125 s^2 - 35 850 a s^2 + 14 225 a^2 s^2 - 8950 m1 s^2 + 7200 a m1 s^2 +
  600 m11 s^2 + 325 m2 s^2 + 600 (2 m11 + m2) s^2 + 2975 s^3 - 2400 a s^3 - 600 m1 s^3 + 150 s^4
```

In[59]:= **d4 = Expand[%58]**

Out[59]=

$$90625 - 296250\,a + 354500\,a^2 - 183750\,a^3 + 34875\,a^4 - 73750\,m1 + 179250\,a\,m1 - 142250\,a^2\,m1 + 36750\,a^3\,m1 + 44750\,m11 - 72000\,a\,m11 + 28450\,a^2\,m11 - 18000\,m111 + 14400\,a\,m111 + 3600\,m1111 + 23000\,m2 - 36750\,a\,m2 + 14450\,a^2\,m2 - 9250\,m21 + 7350\,a\,m21 + 1850\,m211 + 950\,m22 - 3250\,m3 + 2550\,a\,m3 + 650\,m31 + 175\,m4 + 73125\,s - 178500\,a\,s + 142025\,a^2\,s - 36750\,a^3\,s - 44500\,m1\,s + 71850\,a\,m1\,s - 28450\,a^2\,m1\,s + 17950\,m11\,s - 14400\,a\,m11\,s - 3600\,m111\,s + 9225\,m2\,s - 7350\,a\,m2\,s - 1850\,m21\,s - 650\,m3\,s + 22125\,s^2 - 35850\,a\,s^2 + 14225\,a^2\,s^2 - 8950\,m1\,s^2 + 7200\,a\,m1\,s^2 + 1800\,m11\,s^2 + 925\,m2\,s^2 + 2975\,s^3 - 2400\,a\,s^3 - 600\,m1\,s^3 + 150\,s^4$$

In[60]:= **d5 = Expand[(1 / 5) * (4 * d3 - d4)]**

Out[60]=

$$20375 - 67850\,a + 83000\,a^2 - 44150\,a^3 + 8625\,a^4 - 16650\,m1 + 41170\,a\,m1 - 33350\,a^2\,m1 + 8830\,a^3\,m1 + 10130\,m11 - 16560\,a\,m11 + 6670\,a^2\,m11 - 4080\,m111 + 3312\,a\,m111 + 816\,m1111 + 5240\,m2 - 8510\,a\,m2 + 3410\,a^2\,m2 - 2110\,m21 + 1702\,a\,m21 + 422\,m211 + 218\,m22 - 750\,m3 + 598\,a\,m3 + 150\,m31 + 41\,m4 + 16475\,s - 40940\,a\,s + 33275\,a^2\,s - 8830\,a^3\,s - 10060\,m1\,s + 16514\,a\,m1\,s - 6670\,a^2\,m1\,s + 4066\,m11\,s - 3312\,a\,m11\,s - 816\,m111\,s + 2103\,m2\,s - 1702\,a\,m2\,s - 422\,m21\,s - 150\,m3\,s + 4995\,s^2 - 8234\,a\,s^2 + 3335\,a^2\,s^2 - 2026\,m1\,s^2 + 1656\,a\,m1\,s^2 + 408\,m11\,s^2 + 211\,m2\,s^2 + 673\,s^3 - 552\,a\,s^3 - 136\,m1\,s^3 + 34\,s^4$$

(*Calculations of (64/d)c_2(Z) using Lemma 3.1(iii)*)

In[61]:= **u = (3 / 2) * (5 * a - 5 + m1 - s)**

Out[61]=

$$\frac{3}{2}\left(-5 + 5\,a + m1 - s\right)$$

In[62]:= **q = m1 - s - 5**

Out[62]=

$$-5 + m1 - s$$

In[63]:= **Expand[(q + 2 * u) * d2]**

Out[63]=

$$38500 - 132875\,a + 169500\,a^2 - 94625\,a^3 + 19500\,a^4 - 31400\,m1 + 80575\,a\,m1 - 68100\,a^2\,m1 + 18925\,a^3\,m1 + 4740\,m1^2 - 8400\,a\,m1^2 + 3660\,a^2\,m1^2 + 9600\,m11 - 15600\,a\,m11 + 6300\,a^2\,m11 - 1920\,m1\,m11 + 1680\,a\,m1\,m11 - 1920\,m111 + 1440\,a\,m111 + 384\,m1\,m111 + 5100\,m2 - 8225\,a\,m2 + 3300\,a^2\,m2 - 1020\,m1\,m2 + 880\,a\,m1\,m2 - 1020\,m21 + 765\,a\,m21 + 204\,m1\,m21 - 380\,m3 + 285\,a\,m3 + 76\,m1\,m3 + 31100\,s - 80150\,a\,s + 67950\,a^2\,s - 18925\,a^3\,s - 18960\,m1\,s + 32315\,a\,m1\,s - 13620\,a^2\,m1\,s + 1908\,m1^2\,s - 1680\,a\,m1^2\,s + 3840\,m11\,s - 3120\,a\,m11\,s - 384\,m1\,m11\,s - 384\,m111\,s + 2040\,m2\,s - 1645\,a\,m2\,s - 204\,m1\,m2\,s - 204\,m21\,s - 76\,m3\,s + 9420\,s^2 - 16115\,a\,s^2 + 6810\,a^2\,s^2 - 3816\,m1\,s^2 + 3240\,a\,m1\,s^2 + 192\,m1^2\,s^2 + 384\,m11\,s^2 + 204\,m2\,s^2 + 1268\,s^3 - 1080\,a\,s^3 - 256\,m1\,s^3 + 64\,s^4$$

In[64]:=
```
%63 /. {(m1)^2 → m2 + 2 * m11, (m1)^3 → m3 + 3 * m21 + 6 * m111, (m1)^4 → m4 + 4 * m31 +
  6 * m22 + 12 * m211 + 24 * m1111, m1 * m11 → m21 + 3 * m111, (m1)^2 * m11 → m31 + 2 * m22 +
  5 * m211 + 12 * m1111, (m11)^2 → m22 + 2 * m211 + 6 * m1111, m1 * m3 → m4 + m31, m1 *
  m21 → m31 + 2 * m22 + 2 * m211, m1 * m111 → m211 + 4 * m1111, m1 * m2 → m3 + m21, (m1)^
  2 * m2 → m4 + 2 * m31 + 2 * m22 + 2 * m211, (m2)^2 → m4 + 2 * m22, m2 * m11 → m31 + m211}
```

Out[64]=

$$\begin{gathered}38\,500 - 132\,875\,a + 169\,500\,a^2 - 94\,625\,a^3 + 19\,500\,a^4 - 31\,400\,m1 + 80\,575\,a\,m1 - 68\,100\,a^2\,m1 + \\ 18\,925\,a^3\,m1 + 9600\,m11 - 15\,600\,a\,m11 + 6300\,a^2\,m11 - 1920\,m111 + 1440\,a\,m111 + 5100\,m2 - \\ 8225\,a\,m2 + 3300\,a^2\,m2 + 4740\,(2\,m11 + m2) - 8400\,a\,(2\,m11 + m2) + 3660\,a^2\,(2\,m11 + m2) - 1020\,m21 + \\ 765\,a\,m21 - 1920\,(3\,m111 + m21) + 1680\,a\,(3\,m111 + m21) + 384\,(4\,m1111 + m211) - 380\,m3 + 285\,a\,m3 - \\ 1020\,(m21 + m3) + 880\,a\,(m21 + m3) + 204\,(2\,m211 + 2\,m22 + m31) + 76\,(m31 + m4) + 31\,100\,s - 80\,150\,a\,s + \\ 67\,950\,a^2\,s - 18\,925\,a^3\,s - 18\,960\,m1\,s + 32\,315\,a\,m1\,s - 13\,620\,a^2\,m1\,s + 3840\,m11\,s - 3120\,a\,m11\,s - \\ 384\,m111\,s + 2040\,m2\,s - 1645\,a\,m2\,s + 1908\,(2\,m11 + m2)\,s - 1680\,a\,(2\,m11 + m2)\,s - 204\,m21\,s - \\ 384\,(3\,m111 + m21)\,s - 76\,m3\,s - 204\,(m21 + m3)\,s + 9420\,s^2 - 16\,115\,a\,s^2 + 6810\,a^2\,s^2 - 3816\,m1\,s^2 + \\ 3240\,a\,m1\,s^2 + 384\,m11\,s^2 + 204\,m2\,s^2 + 192\,(2\,m11 + m2)\,s^2 + 1268\,s^3 - 1080\,a\,s^3 - 256\,m1\,s^3 + 64\,s^4\end{gathered}$$

In[65]:=
```
d6 = Expand[%64]
```

Out[65]=

$$\begin{gathered}38\,500 - 132\,875\,a + 169\,500\,a^2 - 94\,625\,a^3 + 19\,500\,a^4 - 31\,400\,m1 + 80\,575\,a\,m1 - 68\,100\,a^2\,m1 + \\ 18\,925\,a^3\,m1 + 19\,080\,m11 - 32\,400\,a\,m11 + 13\,620\,a^2\,m11 - 7680\,m111 + 6480\,a\,m111 + \\ 1536\,m1111 + 9840\,m2 - 16\,625\,a\,m2 + 6960\,a^2\,m2 - 3960\,m21 + 3325\,a\,m21 + 792\,m211 + 408\,m22 - \\ 1400\,m3 + 1165\,a\,m3 + 280\,m31 + 76\,m4 + 31\,100\,s - 80\,150\,a\,s + 67\,950\,a^2\,s - 18\,925\,a^3\,s - \\ 18\,960\,m1\,s + 32\,315\,a\,m1\,s - 13\,620\,a^2\,m1\,s + 7656\,m11\,s - 6480\,a\,m11\,s - 1536\,m111\,s + \\ 3948\,m2\,s - 3325\,a\,m2\,s - 792\,m21\,s - 280\,m3\,s + 9420\,s^2 - 16\,115\,a\,s^2 + 6810\,a^2\,s^2 - \\ 3816\,m1\,s^2 + 3240\,a\,m1\,s^2 + 768\,m11\,s^2 + 396\,m2\,s^2 + 1268\,s^3 - 1080\,a\,s^3 - 256\,m1\,s^3 + 64\,s^4\end{gathered}$$

In[66]:=
```
Expand[8 * (Binomial[s + 5, 2] + m1 * q - m11 - (1 / 8) * dul - u^2 - q^2 - 2 * q * u)]
```

Out[66]=

$$-1315 + 1800\,a - 605\,a^2 + 520\,m1 - 360\,a\,m1 - 49\,m1^2 - 6\,m11 - 523\,s + 360\,a\,s + 104\,m1\,s - 52\,s^2$$

In[67]:=
```
Expand[%66 * d1]
```

Out[67]=

$$\begin{gathered}-190\,675 + 655\,500\,a - 831\,550\,a^2 + 460\,500\,a^3 - 93\,775\,a^4 + 154\,300\,m1 - 395\,100\,a\,m1 + 332\,900\,a^2\,m1 - \\ 92\,100\,a^3\,m1 - 38\,305\,m1^2 + 67\,500\,a\,m1^2 - 29\,195\,a^2\,m1^2 + 2940\,m1^3 - 2940\,a\,m1^3 - 16\,650\,m11 + \\ 23\,400\,a\,m11 - 8190\,a^2\,m11 + 6600\,m1\,m11 - 4680\,a\,m1\,m11 - 588\,m1^2\,m11 - 72\,m11^2 - 9205\,m2 + \\ 12\,600\,a\,m2 - 4235\,a^2\,m2 + 3640\,m1\,m2 - 2520\,a\,m1\,m2 - 343\,m1^2\,m2 - 42\,m11\,m2 - 153\,420\,s + \\ 394\,200\,a\,s - 332\,760\,a^2\,s + 92\,100\,a^3\,s + 92\,920\,m1\,s - 158\,220\,a\,m1\,s + 66\,580\,a^2\,m1\,s - \\ 15\,371\,m1^2\,s + 13\,500\,a\,m1^2\,s + 588\,m1^3\,s - 6630\,m11\,s + 4680\,a\,m11\,s + 1320\,m1\,m11\,s - \\ 3661\,m2\,s + 2520\,a\,m2\,s + 728\,m1\,m2\,s - 46\,287\,s^2 + 79\,020\,a\,s^2 - 33\,290\,a^2\,s^2 + 18\,652\,m1\,s^2 - \\ 15\,840\,a\,m1\,s^2 - 1542\,m1^2\,s^2 - 660\,m11\,s^2 - 364\,m2\,s^2 - 6206\,s^3 + 5280\,a\,s^3 + 1248\,m1\,s^3 - 312\,s^4\end{gathered}$$

In[68]:=
```
%67 /. {(m1)^2 → m2 + 2 * m11, (m1)^3 → m3 + 3 * m21 + 6 * m111, (m1)^4 → m4 + 4 * m31 +
  6 * m22 + 12 * m211 + 24 * m1111, m1 * m11 → m21 + 3 * m111, (m1)^2 * m11 → m31 + 2 * m22 +
  5 * m211 + 12 * m1111, (m11)^2 → m22 + 2 * m211 + 6 * m1111, m1 * m3 → m4 + m31, m1 *
  m21 → m31 + 2 * m22 + 2 * m211, m1 * m111 → m211 + 4 * m1111, m1 * m2 → m3 + m21, (m1)^
  2 * m2 → m4 + 2 * m31 + 2 * m22 + 2 * m211, (m2)^2 → m4 + 2 * m22, m2 * m11 → m31 + m211}
```

Out[68]=

$$-190\,675 + 655\,500\,a - 831\,550\,a^2 + 460\,500\,a^3 - 93\,775\,a^4 + 154\,300\,m1 - 395\,100\,a\,m1 + 332\,900\,a^2\,m1 - 92\,100\,a^3\,m1 - 16\,650\,m11 + 23\,400\,a\,m11 - 8190\,a^2\,m11 - 9205\,m2 + 12\,600\,a\,m2 - 4235\,a^2\,m2 - 38\,305\,(2\,m11 + m2) + 67\,500\,a\,(2\,m11 + m2) - 29\,195\,a^2\,(2\,m11 + m2) + 6600\,(3\,m111 + m21) - 4680\,a\,(3\,m111 + m21) - 72\,(6\,m1111 + 2\,m211 + m22) + 3640\,(m21 + m3) - 2520\,a\,(m21 + m3) + 2940\,(6\,m111 + 3\,m21 + m3) - 2940\,a\,(6\,m111 + 3\,m21 + m3) - 42\,(m211 + m31) - 588\,(12\,m1111 + 5\,m211 + 2\,m22 + m31) - 343\,(2\,m211 + 2\,m22 + 2\,m31 + m4) - 153\,420\,s + 394\,200\,a\,s - 332\,760\,a^2\,s + 92\,100\,a^3\,s + 92\,920\,m1\,s - 158\,220\,a\,m1\,s + 66\,580\,a^2\,m1\,s - 6630\,m11\,s + 4680\,a\,m11\,s - 3661\,m2\,s + 2520\,a\,m2\,s - 15\,371\,(2\,m11 + m2)\,s + 13\,500\,a\,(2\,m11 + m2)\,s + 1320\,(3\,m111 + m21)\,s + 728\,(m21 + m3)\,s + 588\,(6\,m111 + 3\,m21 + m3)\,s - 46\,287\,s^2 + 79\,020\,a\,s^2 - 33\,290\,a^2\,s^2 + 18\,652\,m1\,s^2 - 15\,840\,a\,m1\,s^2 - 660\,m11\,s^2 - 364\,m2\,s^2 - 1542\,(2\,m11 + m2)\,s^2 - 6206\,s^3 + 5280\,a\,s^3 + 1248\,m1\,s^3 - 312\,s^4$$

In[69]:=
```
d7 = Expand[%68]
```

Out[69]=

$$-190\,675 + 655\,500\,a - 831\,550\,a^2 + 460\,500\,a^3 - 93\,775\,a^4 + 154\,300\,m1 - 395\,100\,a\,m1 + 332\,900\,a^2\,m1 - 92\,100\,a^3\,m1 - 93\,260\,m11 + 158\,400\,a\,m11 - 66\,580\,a^2\,m11 + 37\,440\,m111 - 31\,680\,a\,m111 - 7488\,m1111 - 47\,510\,m2 + 80\,100\,a\,m2 - 33\,430\,a^2\,m2 + 19\,060\,m21 - 16\,020\,a\,m21 - 3812\,m211 - 1934\,m22 + 6580\,m3 - 5460\,a\,m3 - 1316\,m31 - 343\,m4 - 153\,420\,s + 394\,200\,a\,s - 332\,760\,a^2\,s + 92\,100\,a^3\,s + 92\,920\,m1\,s - 158\,220\,a\,m1\,s + 66\,580\,a^2\,m1\,s - 37\,372\,m11\,s + 31\,680\,a\,m11\,s + 7488\,m111\,s - 19\,032\,m2\,s + 16\,020\,a\,m2\,s + 3812\,m21\,s + 1316\,m3\,s - 46\,287\,s^2 + 79\,020\,a\,s^2 - 33\,290\,a^2\,s^2 + 18\,652\,m1\,s^2 - 15\,840\,a\,m1\,s^2 - 3744\,m11\,s^2 - 1906\,m2\,s^2 - 6206\,s^3 + 5280\,a\,s^3 + 1248\,m1\,s^3 - 312\,s^4$$

In[70]:=
```
d8 = Expand[8 * d6 + d7]
```

Out[70]=

$$117\,325 - 407\,500\,a + 524\,450\,a^2 - 296\,500\,a^3 + 62\,225\,a^4 - 96\,900\,m1 + 249\,500\,a\,m1 - 211\,900\,a^2\,m1 + 59\,300\,a^3\,m1 + 59\,380\,m11 - 100\,800\,a\,m11 + 42\,380\,a^2\,m11 - 24\,000\,m111 + 20\,160\,a\,m111 + 4800\,m1111 + 31\,210\,m2 - 52\,900\,a\,m2 + 22\,250\,a^2\,m2 - 12\,620\,m21 + 10\,580\,a\,m21 + 2524\,m211 + 1330\,m22 - 4620\,m3 + 3860\,a\,m3 + 924\,m31 + 265\,m4 + 95\,380\,s - 247\,000\,a\,s + 210\,840\,a^2\,s - 59\,300\,a^3\,s - 58\,760\,m1\,s + 100\,300\,a\,m1\,s - 42\,380\,a^2\,m1\,s + 23\,876\,m11\,s - 20\,160\,a\,m11\,s - 4800\,m111\,s + 12\,552\,m2\,s - 10\,580\,a\,m2\,s - 2524\,m21\,s - 924\,m3\,s + 29\,073\,s^2 - 49\,900\,a\,s^2 + 21\,190\,a^2\,s^2 - 11\,876\,m1\,s^2 + 10\,080\,a\,m1\,s^2 + 2400\,m11\,s^2 + 1262\,m2\,s^2 + 3938\,s^3 - 3360\,a\,s^3 - 800\,m1\,s^3 + 200\,s^4$$

```
(*Main relation when X is in P^{4+s}, c.i. of type (
d_1,...,d_s), E Ulrich of rank 3 for (X,O_X(a))*)
```

In[71]:= Expand[10 * d5 + d8]

Out[71]=

$$321075 - 1086000\,a + 1354450\,a^2 - 738000\,a^3 + 148475\,a^4 - 263400\,m1 + 661200\,a\,m1 - 545400\,a^2\,m1 + 147600\,a^3\,m1 + 160680\,m11 - 266400\,a\,m11 + 109080\,a^2\,m11 - 64800\,m111 + 53280\,a\,m111 + 12960\,m1111 + 83610\,m2 - 138000\,a\,m2 + 56350\,a^2\,m2 - 33720\,m21 + 27600\,a\,m21 + 6744\,m211 + 3510\,m22 - 12120\,m3 + 9840\,a\,m3 + 2424\,m31 + 675\,m4 + 260130\,s - 656400\,a\,s + 543590\,a^2\,s - 147600\,a^3\,s - 159360\,m1\,s + 265440\,a\,m1\,s - 109080\,a^2\,m1\,s + 64536\,m11\,s - 53280\,a\,m11\,s - 12960\,m111\,s + 33582\,m2\,s - 27600\,a\,m2\,s - 6744\,m21\,s - 2424\,m3\,s + 79023\,s^2 - 132240\,a\,s^2 + 54540\,a^2\,s^2 - 32136\,m1\,s^2 + 26640\,a\,m1\,s^2 + 6480\,m11\,s^2 + 3372\,m2\,s^2 + 10668\,s^3 - 8880\,a\,s^3 - 2160\,m1\,s^3 + 540\,s^4$$

In[72]:= chisprime = (1 / (64 * 12)) * %71

Out[72]=

$$\frac{1}{768}\left(321075 - 1086000\,a + 1354450\,a^2 - 738000\,a^3 + 148475\,a^4 - 263400\,m1 + 661200\,a\,m1 - 545400\,a^2\,m1 + 147600\,a^3\,m1 + 160680\,m11 - 266400\,a\,m11 + 109080\,a^2\,m11 - 64800\,m111 + 53280\,a\,m111 + 12960\,m1111 + 83610\,m2 - 138000\,a\,m2 + 56350\,a^2\,m2 - 33720\,m21 + 27600\,a\,m21 + 6744\,m211 + 3510\,m22 - 12120\,m3 + 9840\,a\,m3 + 2424\,m31 + 675\,m4 + 260130\,s - 656400\,a\,s + 543590\,a^2\,s - 147600\,a^3\,s - 159360\,m1\,s + 265440\,a\,m1\,s - 109080\,a^2\,m1\,s + 64536\,m11\,s - 53280\,a\,m11\,s - 12960\,m111\,s + 33582\,m2\,s - 27600\,a\,m2\,s - 6744\,m21\,s - 2424\,m3\,s + 79023\,s^2 - 132240\,a\,s^2 + 54540\,a^2\,s^2 - 32136\,m1\,s^2 + 26640\,a\,m1\,s^2 + 6480\,m11\,s^2 + 3372\,m2\,s^2 + 10668\,s^3 - 8880\,a\,s^3 - 2160\,m1\,s^3 + 540\,s^4\right)$$

In[73]:= Expand[chisprime - Fa4s30]

Out[73]=

$$\frac{7}{256} - \frac{25\,a^2}{384} + \frac{29\,a^4}{768} - \frac{5\,m2}{384} + \frac{5\,a^2\,m2}{384} + \frac{m22}{384} + \frac{3\,m4}{1280} + \frac{23\,s}{1920} - \frac{5\,a^2\,s}{384} - \frac{m2\,s}{384} + \frac{s^2}{768}$$

In[74]:= $\text{Factor}\left[\frac{7}{256} - \frac{25\,a^2}{384} + \frac{29\,a^4}{768} - \frac{5\,m2}{384} + \frac{5\,a^2\,m2}{384} + \frac{m22}{384} + \frac{3\,m4}{1280} + \frac{23\,s}{1920} - \frac{5\,a^2\,s}{384} - \frac{m2\,s}{384} + \frac{s^2}{768}\right]$

Out[74]=

$$\frac{105 - 250\,a^2 + 145\,a^4 - 50\,m2 + 50\,a^2\,m2 + 10\,m22 + 9\,m4 + 46\,s - 50\,a^2\,s - 10\,m2\,s + 5\,s^2}{3840}$$